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Aims and Scope
The journal publishes papers from worldwide in the field of instrumentation, measurement, and control. Its scope encompasses cutting-edge research and development, education and industrial applications. The journal publishes peer-reviewed papers designed to appeal to both researchers and practitioners. It presents up-to-date coverage of the latest developments, offering a unique interdisciplinary perspective. It also includes invited papers, book reviews, conference notices, calls for papers, and announcements of new publications and special issues. The journal covers - but not limited to: All Aspects of Control Theory and its Applications (Linear, Nonlinear, Robust, State Space and Multivariable, Self-Tuning, Adaptive, Optimal, and others), System Identification and Parameter Estimation, and Mathematical Modeling and Simulation, Intelligent Systems and Applications (fuzzy logic, neural networks, genetic algorithms, and others), Machine Vision and Pattern Recognition, Mechatronics and Robotics, and Advanced Manufacturing Systems, Sensors, Measuring Instruments, and Signal Processing, Computing for Measurement, Control and Automation, and Industrial Networks and Communication protocols (FieldBuses, OPC, and others). Papers may be from the following categories: Design: Present a complete "how-to" guide. Connect design procedures to first principles. Explicitly state necessary heuristics and limits of applicability. Provide evidence that the procedures are practicable. Analysis: Clearly develop a fundamental, theoretical analysis of a practice-relevant issue. Explicitly state implications and recommendations for its application. Provide credible examples. Application: Present the results of new (or under-utilized) techniques or novel applications. Provide a complete description of results, including pilot- or plant-scale experimental data, and a revelation of heuristics and shortcomings.

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WELCOME TO THE MEDITERRANEAN JOURNAL OF MEASUREMENT AND CONTROL

We would like to welcome you all to the third volume - first issue - of The Mediterranean Journal of Measurement and Control. This issue of the journal, comprising of five papers, is dedicated to the latest research work being done in the analyses of modeling and control of dynamic systems.

The first paper by Xiao deals with the problem of a jump linear-quadratic-Gaussian (JLQG) control of continuous-time Markov jump systems with uncertain second-order statistical properties. Uncertainty is modeled by allowing process and observation noise spectral density matrices to vary arbitrarily within given classes. The upper bounds of the perturbation to the noise covariances matrix are given based on the guaranteed control performance, and a robust minimax JLQG regulator is therefore adopted under the worst conditions. Not only can this method minimizes the worst quadratic object performance function of the uncertainty, but the control performance index can be guaranteed to be within a given freedom.

The second paper by Mahmoud deals with delay-independent and delay-dependent robust stabilization schemes that are established for a class of continuous-time systems with state-delays and norm-bounded uncertainties against controller gain variations. State-feedback resilient adaptive controllers are constructed for the case of known gain perturbation bounds and then extended to accommodate unknown norm-bounded perturbations. All the developed results are conveniently expressed in linear matrix inequalities (LMIs) format.

The third paper by Röbenack proposes a new technique that uses an observer to estimate the current input into a neuron whose voltage is measured electrophysiologically. As a by-product, one also obtains informations about the gating variables of the ionic channels. We prove the global convergence of the observer for all voltage-gated ion channel models within the Hodgkin-Huxley formalism. The current observer can be implemented either offline or concurrently with the recording.

The fourth paper by Lalili presents an algorithm for the space vector pulse width modulation (SVPWM) applied to three-level diode clamped inverter. In this algorithm, the space vector diagram of the three-level inverter is decomposed into six space vector diagrams of two-level inverters. This idea allows to generalize the two-level SVPWM algorithm into the case of three-level inverter. The redundant vectors of the space vector diagram of three-level inverter are used to control its neutral point potential using a closed loop.

Finally, the fifth paper by Fawzy deals with the problem of designing a global robust model reference adaptive output feedback tracking control for SISO nonlinear systems containing a vector of unknown constant parameters; entering linearly and subject to bounded disturbances with unknown bound. Furthermore, there is no a priori knowledge assumed on the sign of the high frequency gain. A Nussbaum gain is introduced in the global adaptive algorithm to ensure that the output tracks any bounded reference signal.

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Dr. Mohamed H. Mahmoud
Editor-in-Chief

ROBUST JLQG REGULATOR OF GUARANTEED CONTROL PERFORMANCE WITH UNCERTAIN NOISE

X. Xiao, H. Xi *, J. Zhu, H. Ji
Department of Automation, University of Science and Technology of China, P. R. China

ABSTRACT

The problem of a jump linear-quadratic-Gaussian (JLQG) control of continuous-time Markov jump systems with uncertain second-order statistical properties is considered. Uncertainty is modeled by allowing process and observation noise spectral density matrices to vary arbitrarily within given classes. The upper bounds of the perturbation to the noise covariances matrix are given based on the guaranteed control performance, and a robust minimax JLQG regulator is therefore adopted under the worst conditions. Not only can this method minimizes the worst quadratic object performance function of the uncertainty, but the control performance index can be guaranteed to be within a given freedom. Finally, two numerical examples are included to demonstrate the performance of this method.

Keywords

Markov Jump Systems, Uncertain Noise, Robust JLQG Regulator, Perturbation Bound.

1. INTRODUCTION

The study of the linear-quadratic-Gaussian (LQG) optimal control problem for finite dimensional systems keeps attracting considerable attentions due to demands from practical dynamical processes in mechanics, chemical process, automotive systems and electrical circuit systems and others. A well-known property of the LQG optimal control problem is that the optimal regulator, synthesized by the LQ optimal technique, is generated from the estimated state which is the output of the Kalman filter. The standard LQG optimal control design for a linear stochastic dynamic system requires not only an accurate description of the statistical characteristic of noise signal but also an exact system model. Nevertheless, in many actual problems, the noise covariances are hardly known. Consequently, the standard LQG regulator may not be robust against modeling uncertainty and disturbances. Therefore, the study of robust LQG optimal problem for those systems whose noise covariances are known only to be within some classes is of practical importance and has been attracting more interest over the past several decades. A useful approach is to use a game-theoretic formulation with which one minimizes the worst performance stimulated by uncertain factors and this approach has been used successfully to design the minimax robust controller for linear systems with uncertain noise. And some representative results of LQG control for linear system with uncertain noise have been addressed in [1-3].

On the other hand, many physical systems have variable structures subject to random abrupt changes, which may result from abrupt phenomena such as random failures and repairs of the components, changes in the interconnections of subsystems, sudden environmental changes, and others. A system with this character may be modeled as a hybrid one; that is, the state space of the system contains both discrete and continuous states. A special class of hybrid systems is the so-called Markov jump systems. In the past three decades, Markov jump systems have been extensively studied, and a lot of achievements have been made on control design, filtering and stability analysis, see [4-16]. With the problem of JLQG control, there also exists a vast literature, see [12-16]. We note that, a common feature of those studies is that the exogenous input signals are all assumed to be Gaussian with known statistic. Considering that there exist uncertain factors in many actual problems, for continuous-time Markov jump linear systems, the study of a robust JLQG control approach under uncertain noise is of practical importance. However, to date this problem has not yet fully investigated.

In this paper, we are interested in robust JLQG regulator design for continuous-time Markov jump systems with uncertain noise. Uncertainty is modeled by allowing process and observation noise spectral density matrices to vary arbitrarily within given classes. Firstly, we give a perturbation upper bound on uncertain noise covariances in which the deviation of the control performance index is guaranteed to be within a certain bound. Secondly, the worst performance yielded by the noise uncertainty can be minimized by a minimax robust JLQG regulator. The paper is organized as follows: the problem of robust JLQG regulator for Markov jump systems is formulated in Section 2. In Section 3, a correspondence relation between the control performance index and the perturbation noise covariances is demonstrated, and a simple and direct way is given to derive a perturbation upper bound on uncertain noise covariances. The minimax robust JLQG regulator, based on Game Theory, is formulated in Section 4. Finally, two numerical examples are included.

2. PROBLEM FORMULATION AND PRELIMINARIES

In this section, the problem of robust JLQG regulator for Markov jump systems with uncertain noise is formulated firstly. For a class of Markov jump linear systems, when the

*Corresponding author: *E-mail:* xiaoxiaobo@ustc.edu

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process and observation noise spectral density matrices vary arbitrarily within given classes, the control performance index may deviate from the ideal value. Therefore, how to guarantee the robustness of control performance for the JLQG regulator is a question to be solved. Secondly, to illustrate the relationship between the control performance index and the uncertain noise, we develop the equivalent form of the control performance index.

Consider the following class of dynamical systems in a fixed complete probability space $(\Omega, F, \mathbf{P})$:

$$\begin{cases} \dot{x}(t) = A(r_t)x(t) + B(r_t)u(t) + \omega_t^0 \\ y(t) = C(r_t)x(t) + \upsilon_t^0 \end{cases} \quad (1)$$

where $x(t) \in \mathbf{R}^n$ is the system state, $y(t) \in \mathbf{R}^m$ is the measurement, $u(t) \in \mathbf{R}^l$ is the control input. The form process $\{r_t, t \geq 0\}$ is a time homogeneous Markov process with right continuous trajectories and taking values in finite state space $S = \{1, \ldots, N\}$ with transition probability matrix $\Pi = (\pi_{ij})_{N \times N}(i, j \in S)$ given by:

$$P\{r_{(t+\Delta)} = j \mid r_t = i\} = \begin{cases} \pi_{ij}\Delta + o(\Delta) & i \neq j \\ 1 + \pi_{ii}\Delta + o(\Delta) & i = j \end{cases} \quad (2)$$

where $\lim_{\Delta \to 0} \frac{o(\Delta)}{\Delta} = 0, (\Delta > 0)$,

π_{ij} is the transition rate from mode i at time t to mode j at time $t + \Delta$, and $\pi_{ii} = -\sum_{j \neq i} \pi_{ij}$, $(\pi_{ij} \geq 0, j \neq i)$. For any $i \in S$, $A(r_t) \in \mathbf{R}^{n \times n}$, $B(r_t) \in \mathbf{R}^{n \times l}$, $C(r_t) \in \mathbf{R}^{m \times n}$ are known constant matrices. There $\omega_t^0 \in \mathbf{R}^n$, $\upsilon_t^0 \in \mathbf{R}^m$ are the process and measurement noises respectively. And we should make the following assumptions on the process noise ω_t^0 and measurement noise υ_t^0.

Assumption 1 *For all* $t \geq 0$, $s \geq 0$ *and* $i \in S$:

$$\begin{aligned} &(1) E[\omega_t^0] = 0, \qquad E[\upsilon_t^0] = 0; \\ &(2) Cov[\omega_t^0, \omega_s^0] = W^0\delta(t-s) = (W + \Delta W)\delta(t-s) \\ &\qquad W \geq 0, \Delta W \geq 0; \\ &(3) Cov[\upsilon_t^0, \upsilon_s^0] = V^0\delta(t-s) = (V + \Delta V)\delta(t-s) \\ &\qquad V > 0, \Delta V \geq 0; \\ &(4) E[\omega_t \upsilon_t^T] = 0. \end{aligned}$$

where $W^0 \in \mathbf{R}^{n \times n}$, $V^0 \in \mathbf{R}^{m \times m}$. W and V are the known nominal noise covariance matrices; ΔW and ΔV are the unknown but bounded perturbation noise covariance matrices. In assumption 1, the notation $W \geq 0$ (respectively, $W > 0$) means that W is positive semi-definite (respectively positive). $\delta(\cdot)$ is the Dirac function.

The objective of the JLQG control is to choose $u(t)$, for $t \in [0, \infty)$ to minimize the time average quadratic cost

$$\begin{aligned} \mathbf{J}(u, W^0, V^0, i_0) = \lim_{T \to \infty} E\{\frac{1}{T}\int_0^T [x^T(t)Q(r_t)x(t) \\ + u^T(t)R(r_t)u(t)]dt \mid x_0, r_0 = i_0, t_0 = 0\} \end{aligned} \quad (3)$$

where $Q(r_t) \geq 0$ and $R(r_t) \geq 0$.

To simplify the denotation, we use the subscript i denote r_t, such as $A(r_t = i) = A_i$.

The next definitions of stochastic stability and stochastic detectability of system (1) is given in [5] and [4].

Definition 1 *We say that the noise-free system (1) (or the triplet* $(A_i, B_i, \Pi, i \in S)$*) is stochastically stabilizable, if for every initial state* (x_0, r_0)*, there exists a linear feedback control law* $u(t) = -L_i x(t)$, $i \in S$ *such that there exists a symmetric positive definite matrix M satisfying*

$$\lim_{T \to \infty} E\{\int_0^T x^T(t, x_0, r_0, u)x(t, x_0, r_0, u)dt \mid x_0, r_0\} \leq x_0^T M x_0$$

Definition 2 *We say that the system* $(A_i, B_i, \Pi, i \in S)$ *is stochastically detectable if its dual* $(A_i^T, B_i^T, \Pi, i \in S)$ *is stochastically stabilizable.*

Assumption 2 *For Markov jump systems (1) with the control object function (3):*

(1) *The triplet* $(A_i, C_i, \Pi, i \in S)$ *is stochastically detectable;*

(2) *The triplet* $A_i, W^{1/2}, \Pi, i \in S$ *is stochastically stabilizable;*

(3) *The triplet* $(A_i, B_i, \Pi, i \in S)$ *is stochastically stabilizable;*

(4) *The triplet* $A_i, Q_i^{1/2}, \Pi, i \in S$ *is stochastically detectable.*

By [4], we know that, under the condition of **Assumption 2**, when the Markov jump systems (1) only include the known nominal noise W and V, the solution to the JLQG regulator problem (1)-(3) is given by the feedback system

$$u(t) = -G_i\hat{x}, \quad t \geq 0 \quad (4)$$

$$G_i = R_i^{-1}B_i^T P_i \quad (5)$$

$$0 = P_i A_i + A_i^T P_i - P_i B_i R_i^{-1} B_i^T P_i + \sum_{j=1}^N \pi_{ij} P_j + Q_i \quad (6)$$

$$\dot{\hat{x}}(t) = A_i x(t) + B_i u(t) + H_i[y(t) - C_i\hat{x}(t)] \quad (7)$$

$$H_i = P_{2i} C_i^T V^{-1} \quad (8)$$

$$0 = A_i P_{2i} + P_{2i} A_i^T - P_{2i} C_i^T V^{-1} C_i P_{2i} + \sum_{j=1}^N \pi_{ij} P_{2j} + W \quad (9)$$

If there exist uncertain noises of the process and measurement (that is, $\Delta W \neq 0$, $\Delta V \neq 0$), the covariance matrices of noise become W^0 and V^0. Let us consider the case that the regulator has been designed with nominal noises covariances instead of the actual noise covariances, that is, we still use the filter (7) to design the regulator.

Then the actual estimation error covariance matrix P_{2i}^0, $i \in S$ satisfy the following coupled *Lyapunov* equation:

$$0 = (A_i - H_iC_i)P_{2i}^0 + P_{2i}^0(A_i - H_iC_i)^T + H_iV^0H_i^T + \sum_{j=1}^{N} \pi_{ij}P_{2j}^0 + W^0 \tag{10}$$

Let us define

$$e(t) = x(t) - \hat{x}(t) \tag{11}$$

Then, on combining (1), (4) and (7) with (11), we have

$$\begin{bmatrix} \dot{x}(t) \\ \dot{e}(t) \end{bmatrix} = \begin{bmatrix} A_i - B_iG_i & B_iG_i \\ 0 & A_i - H_iC_i \end{bmatrix} \begin{bmatrix} x(t) \\ e(t) \end{bmatrix} + \begin{bmatrix} W \\ W - H_iV \end{bmatrix} \tag{12}$$

Hence, the cost can be written as

$$\mathbf{J}_{i_0}(u, W^0, V^0) = \lim_{t\to\infty} E\{x^T(t)Q_ix(t) + u^T(t)R_iu(t)\} \tag{13}$$

Substituting (4) and (11) into (3) gives

$$\begin{aligned} \mathbf{J}_{i_0}(u, W^0, V^0) = \lim_{t\to\infty} E\{&x^T(t)(Q_i + G_i^TR_iG_i)x(t) \\ &-x^T(t)G_i^TR_iG_ie(t) - e^T(t)G_i^TR_iG_ix(t) \\ &+e^T(t)G_i^TR_iG_ie(t)\} \end{aligned} \tag{14}$$

Define

$$\bar{x}(t) = \begin{bmatrix} x(t) \\ e(t) \end{bmatrix}, \quad Z = \lim_{t\to\infty} E\{\begin{bmatrix} x(t) \\ e(t) \end{bmatrix}\begin{bmatrix} x(t) \\ e(t) \end{bmatrix}^T\}$$

$$\bar{Q}_i = \begin{bmatrix} Q_i + G_i^TR_iG_i & -G_i^TR_iG_i \\ -G_i^TR_iG_i & G_i^TR_iG_i \end{bmatrix}$$

$$\bar{A}_i = \begin{bmatrix} A_i - B_iG_i & B_iG_i \\ 0 & A_i - H_iC_i \end{bmatrix}, \quad \bar{M}_i = \begin{bmatrix} W & W \\ W & W + H_iVH_i^T \end{bmatrix}$$

For $r_t = i$, $i \in S$, let $\bar{x}(t)\bar{x}^T(t) = X(t)$, then

$$E\{\bar{x}(t)\bar{x}^T(t)\} = \sum_{i=1}^{N} \pi_{ij}X_j(t), \quad \forall t \geq 0$$

where $X_j(t) \triangleq X(t)$ when $r_t = j$

Define

$$Z_i(t) = \sum_{i=1}^{N} \pi_{ij}X_j(t)$$

It can be easily shown (see, for example, [7]) that $Z_i(t) = Z_i^T(t) \geq 0$, $\forall t \geq 0$.

Letting $Z_i = \lim_{t\to\infty} Z_i(t)$, then Z_i satisfies

$$\bar{A}_i^TZ_i + Z_i\bar{A}_i + \sum_{j=1}^{N} \pi_{ij}Z_j + \bar{M}_i = 0 \tag{15}$$

As the method of proof of Theorem 3.5 in [4], averaging over the initial state and permuting integration and expectation, then the cost (14) can be written

$$\mathbf{J}_{i_0}(u, W^0, V^0) = tr[\bar{Q}_iZ_i] \tag{16}$$

where $tr[]$ is the trace of matrix. We note that, Z_i can be rewritten as follows

$$Z_i = \int_0^\infty \exp(L_is)^T \cdot (\bar{M}_i + \sum_{j\in S, j\neq i} \pi_{ij}Z_j) \cdot \exp(L_is)ds \tag{17}$$

where $L_i = \bar{A}_i + \frac{1}{2}\pi_{ij}I$.

In [5], it has been shown that the matrices $\bar{A}_i + \frac{1}{2}\pi_{ij}I$, $i \in j$ are stable. Hence, (16) becomes

$$\begin{aligned} \mathbf{J}_{i_0}(u, W^0, V^0) = tr[\bar{Q}_i\int_0^\infty &\exp(L_is)^T \cdot (\bar{M}_i \\ &+ \sum_{j\in S, j\neq i} \pi_{ij}Z_j) \cdot \exp(L_is)ds] \end{aligned} \tag{18}$$

With few manipulations, (18) can be rewritten as

$$\begin{aligned} \mathbf{J}_{i_0}(u, W^0, V^0) &= tr[(\bar{M}_i + \sum_{j\in S, j\neq i} \pi_{ij}Z_j) \\ &\quad \cdot \int_0^\infty \exp(L_is)^T \cdot \bar{Q}_i \cdot \exp(L_is)ds] \\ &= tr[(\bar{M}_i + \sum_{j\in S, j\neq i} \pi_{ij}Z_j)\bar{K}_i] \end{aligned} \tag{19}$$

where

$$\bar{A}_i^T\bar{K}_i + \bar{K}_i\bar{A}_i + \bar{Q}_i + \sum_{j=1}^{N} \pi_{ij}\bar{K}_j = 0 \tag{20}$$

Let

$$\overline{K}_i = \begin{bmatrix} K_{i1} & K_{i2} \\ K_{i2}^T & K_{i3} \end{bmatrix}$$

Then we can get three equations:

$$0 = (A_i - B_iG_i)^T K_{i1} + K_{i1}(A_i - B_iG_i) + \sum_{j=1}^{N} \pi_{ij} K_{j1} + Q_i + G_i^T R_i G_i \quad (21)$$

$$0 = (A_i - B_iG_i)^T K_{i2} + K_{i2}(A_i - B_iG_i) + \sum_{j=1}^{N} \pi_{ij} K_{j2} + K_{i1} B_i G_i - G_i^T R_i G_i \quad (22)$$

$$0 = (A_i - H_iC_i)^T K_{i3} + K_{i3}(A_i - H_iC_i) + \sum_{j=1}^{N} \pi_{ij} K_{j3} + K_{i2}^T B_i G_i + G_i^T B_i K_{i2} + G_i^T R_i G_i \quad (23)$$

Combining (21) with (6) gives

$$K_{i1} = P_i \quad (24)$$

Combining (24) with (22) gives

$$K_{i2} = 0$$

And (23) becomes

$$0 = (A_i - H_iC_i)^T K_{i3} + K_{i3}(A_i - H_iC_i) + \sum_{j=1}^{N} \pi_{ij} K_{j3} + G_i^T R_i G_i \quad (25)$$

Then average quadratic cost becomes

$$\mathbf{J}_{i_0}(u, W^0, V^0) = tr[W^0(P_i + K_{i3})] + tr[V^0 H_i K_{i3} H_i^T] + tr[(\sum_{j\in S, j\neq i} \pi_{ij} Z_j)\overline{K}_i] \quad (26)$$

Now, we define the deviation of the control object index to the ideal value, which is yielded by the perturbation noise covariances, as

$$\begin{aligned}\Delta\mathbf{J}_{i_0}(u, \Delta W, \Delta V) &= \mathbf{J}_{i_0}(u, W^0, V^0) - \mathbf{J}_{i_0}(u, W, V) \\ &= tr[\Delta W(P_i + K_{i3})] + tr[(\Delta V H_i K_{i3} H_i^T)] \\ &\leq \gamma_i \qquad \forall i \in S\end{aligned} \quad (27)$$

where $\gamma_i, i \in S$ is an arbitrary given upper bound of the deviation. Obviously, $\Delta\mathbf{J}_{i_0}(u, \Delta W, \Delta V)$ is linear with respect to ΔW and ΔV.

Our purpose is to find a set Ω^* associated with a perturbation upper bound on uncertain noise covariances such that for $(\Delta W, \Delta V) \in \Omega^*$, the deviation of the control object index is guaranteed to be within a given bound $\gamma_i, i \in S$. And the worst performance yielded by the noise uncertainty can be minimized by a robust JLQG regulator.

3. PERTURBATION UPPER BOUND

In this section, we first give some important properties of the deviation of the control performance index. Then we develop the method to find the set Ω^* associated with a perturbation upper bound on uncertain noise covariances.

Suppose

$$\Omega = \{(\Delta W, \Delta V) : 0 \leq \Delta W \leq \Delta W^M, 0 \leq \Delta V \leq \Delta V^M\}$$

to be a compact convex set. It is clear that the $\Delta\mathbf{J}_{i_0}(u, \Delta W, \Delta V)$: $\Omega \to \mathbf{R}$ is a linear mapping for $(\Delta W, \Delta V) \in \Omega$. Some important as well as practical properties of $\Delta\mathbf{J}_{i_0}(u, \Delta W, \Delta V)$ are contained in the following lemmas.

Lemma 1 *For $(\Delta W_j, \Delta V_j) \in \Omega (j = 1, 2)$, if $\Delta W_1 \leq \Delta W_2$ and $\Delta V_1 \leq \Delta V_2$, then*

$$\Delta\mathbf{J}_{i_0}(u, \Delta W_1, \Delta V_1) \leq \Delta\mathbf{J}_{i_0}(u, \Delta W_2, \Delta V_2)$$

Proof. *since $\Delta W_1 \leq \Delta W_2$ and $\Delta V_1 \leq \Delta V_2$, it follows that $\tilde{\Delta W} = \Delta W_2 - \Delta W_1 \geq 0$ and $\tilde{\Delta V} = \Delta V_2 - \Delta V_1 \geq 0$. By equation (27), we have*

$$\begin{aligned}&\Delta\mathbf{J}_{i_0}(u, \Delta W_2, \Delta V_2) - \Delta\mathbf{J}_{i_0}(u, \Delta W_1, \Delta V_1) \\ &= tr[\Delta W_2(P_i + K_{i3})] + tr[(\Delta V_2 H_i K_{i3} H_i^T)] \\ &\quad - tr[\Delta W_1(P_i + K_{i3})] + tr[(\Delta V_1 H_i K_{i3} H_i^T)] \\ &= tr[\Delta\tilde{W}(P_i + K_{i3})] + tr[(\Delta\tilde{V} H_i K_{i3} H_i^T)] \\ &\leq 0\end{aligned}$$

Lemma 2 *For $(\Delta W_j, \Delta V_j) \in \Omega (j = 1, 2)$, $\alpha \in R$, $0 < \alpha < 1$, if $\Delta W = \alpha\Delta W_1 + (1-\alpha)\Delta W_2$, $\Delta V = \alpha\Delta V_1 + (1-\alpha)\Delta V_2$, then*

$$\begin{aligned}&\Delta\mathbf{J}_{i_0}(u, \Delta W, \Delta V) \\ &\leq max\{\Delta\mathbf{J}_{i_0}(u, \Delta W_1, \Delta V_1), \Delta\mathbf{J}_{i_0}(u, \Delta W_2, \Delta V_2)\}\end{aligned} \quad (28)$$

Proof. *Put $\Delta W = \alpha\Delta W_1 + (1-\alpha)\Delta W_2$ and $\Delta V = \alpha\Delta V_1 + (1-\alpha)\Delta V_2$ into (7), we have*

$$\Delta\mathbf{J}_{i_0}(u,\Delta W,\Delta V)=\alpha\Delta\mathbf{J}(u,\Delta W_1,\Delta V_1)+(1-\alpha)\Delta\mathbf{J}_{i_0}(u,\Delta W_2,\Delta V_2) \quad (29)$$

If $\Delta\mathbf{J}_{i_0}(u,\Delta W_1,\Delta V_1)\geq\Delta\mathbf{J}_{i_0}(u,\Delta W_2,\Delta V_2)$, *rewrite (29) as*

$$\Delta\mathbf{J}_{i_0}(u,\Delta W,\Delta V)=\Delta\mathbf{J}_{i_0}(u,\Delta W_1,\Delta V_1)+(1-\alpha)[\Delta\mathbf{J}_{i_0}(u,\Delta W_2,\Delta V_2)-\Delta\mathbf{J}_{i_0}(u,\Delta W_1,\Delta V_1)]$$

By $0<\alpha<1$, *then*

$$\Delta\mathbf{J}_{i_0}(u,\Delta W,\Delta V)\leq\Delta\mathbf{J}_{i_0}(u,\Delta W_1,\Delta V_1)$$

If $\Delta\mathbf{J}_{i_0}((u,\Delta W_1,\Delta V_1)\leq\Delta\mathbf{J}_{i_0}(u,\Delta W_2,\Delta V_2)$, *rewrite (29) as*

$$\Delta\mathbf{J}_{i_0}(u,\Delta W,\Delta V)=\Delta\mathbf{J}_{i_0}(u,\Delta W_2,\Delta V_2)+\alpha[\Delta\mathbf{J}_{i_0}(u,\Delta W_1,\Delta V_1)-\Delta\mathbf{J}_{i_0}(u,\Delta W_2,\Delta V_2)]$$

and therefore

$$\Delta\mathbf{J}_{i_0}(u,\Delta W,\Delta V)\leq\Delta\mathbf{J}_{i_0}(u,\Delta W_2,\Delta V_2)$$

This implies inequality (28) holding.

Applying the results of **Lemma 1** and **Lemma 2**, we can directly obtain the following theorem.

Theorem 1 *Suppose*

$$\Omega=\{(\Delta W,\Delta V):0\leq\Delta W\leq\Delta W^M,\ 0\leq\Delta V\leq\Delta V^M\}$$

to be a compact convex set, and write

$$\gamma_i=\max_{i\in S,(\Delta W,\Delta V)\in\Omega}\Delta\mathbf{J}_{i_0}(u,\Delta W,\Delta V)$$

then γ_i must be reached by matrix pair $(\Delta W^M,\Delta V^M)$ in Ω for any mode $i\in S$.

Supposing the weights of ω_t^0 and υ_t^0 are independent respectively, that is

$$W^0=W+\Delta W=\begin{pmatrix}\sigma_1^2 & 0 & \cdots & 0\\ 0 & \sigma_2^2 & \cdots & 0\\ & & \ddots & \\ 0 & 0 & \cdots & \sigma_n^2\end{pmatrix}+\begin{pmatrix}\varepsilon_1^0 & 0 & \cdots & 0\\ 0 & \varepsilon_2^0 & \cdots & 0\\ & & \ddots & \\ 0 & 0 & \cdots & \varepsilon_n^0\end{pmatrix} \quad (30)$$

$$V^0=V+\Delta V=\begin{pmatrix}\delta_1^2 & 0 & \cdots & 0\\ 0 & \delta_2^2 & \cdots & 0\\ & & \ddots & \\ 0 & 0 & \cdots & \delta_m^2\end{pmatrix}+\begin{pmatrix}e_1^0 & 0 & \cdots & 0\\ 0 & e_2^0 & \cdots & 0\\ & & \ddots & \\ 0 & 0 & \cdots & e_m^0\end{pmatrix} \quad (31)$$

where $\sigma_s(s=1,2,\ldots,n)$ and $\delta_t(t=1,2,\ldots,m)$ are the covariances of corresponding weight without uncertain noise $(\Delta W\equiv 0,\Delta V\equiv 0)$ respectively, and ε_s^0, e_t^0 are the non-negative perturbation parameters. Noting the s-th (t-th) diagonal element of $W_s(V_t)$ is 1, and the other elements are zero, then

$$\Delta W=\sum_{s=1}^{n}\varepsilon_s^0W_s,\qquad \Delta V=\sum_{t=1}^{m}e_t^0V_t$$

Consequently, (27) can be rewritten as

$$\begin{aligned}\Delta\mathbf{J}_{i_0}(u,\Delta W,\Delta V)&=\sum_{s=1}^{n}\varepsilon_s^0tr[W_s(P_i+K_{i3})]+\sum_{t=1}^{m}e_t^0tr[V_tH_iK_{i3}H_i^T]\\&=\sum_{s=1}^{N}\varepsilon_s^0a_{si}+\sum_{t=1}^{N}e_t^0b_{si}\end{aligned} \quad (32)$$

where

$$a_{si}=tr[W_s(P_i+K_{i3})];\ b_{ti}=tr[V_tH_iK_{i3}H_i^T]$$

Define the maximal perturbation set as follows

$$\Delta W^M=\sum_{s=1}^{n}\varepsilon_sW_s,\quad \varepsilon_s^0\leq\varepsilon_s;\ \Delta V^M=\sum_{t=1}^{m}e_tV_t,\quad e_t^0\leq e_t$$

for $s=1,2,\ldots,n$ and $t=1,2,\ldots,m$. By (27), when the perturbation is maximal, we have

$$\sum_{s=1}^{N}\varepsilon_sa_{si}+\sum_{t=1}^{N}e_tb_{ti}\leq\gamma_i \quad (33)$$

And we also define a class of compact convex sets constrained on (33) as

$$\Omega(\varepsilon_s, e_t) = \{(\Delta W, \Delta V) : 0 \le \Delta W \le \sum_{s=1}^{n} \varepsilon_s W_s,$$
$$0 \le \Delta V \le \sum_{t=1}^{m} e_t V_t\}$$

where $\varepsilon = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n)$ and $e = (e_1, e_2, \ldots, e_m)$ are variable vectors. γ_i, $i \in S$ is an arbitrary given upper bound of the deviation (27).

In consideration of $W_s > 0$ and $V_t > 0$, the search of the maximal set Ω^* in $\Omega(\varepsilon_i, e_t)$ is topologically equivalent to the calculation of the maximal volume set in a class of hypercube set $\Psi(\varepsilon_i, e_t)$ defined on (33), where

$$\Psi(\varepsilon_s, e_t) = \{(\varepsilon_s, e_t) : 0 \le \varepsilon_s^0 \le \varepsilon_s, s = 1, 2 \cdots, n;$$
$$0 \le e_t^0 \le e_t, t = 1, 2 \cdots, m\}$$

This problem is attributed to the following

$$\begin{cases} max \quad \{\varepsilon_1\varepsilon_2 \cdots \varepsilon_n e_1 e_2 \cdots e_m\}, \varepsilon_s \ge 0, s = 1, 2 \cdots, n; \\ \qquad e_t \ge 0, t = 1, 2 \cdots, m. \\ s.t. \quad \sum_{s=1}^{n} \varepsilon_s a_{si} + \sum_{t=1}^{m} e_t b_{ti} \le \gamma_i, \quad i \in S \end{cases} \tag{34}$$

Then (34) can be transformed into an equivalent problem of nonlinear programming constrained by linear inequalities.

$$\begin{cases} min \quad -\{\varepsilon_1\varepsilon_2 \cdots \varepsilon_n e_1 e_2 \cdots e_m\}, \varepsilon_s \ge 0, \\ \qquad s = 1, 2 \cdots, n; e_t \ge 0, t = 1, 2 \cdots, m. \\ s.t. \quad \sum_{s=1}^{n} \varepsilon_s a_{si} + \sum_{t=1}^{m} e_t b_{ti} \le \gamma_i, \quad i \in S \end{cases} \tag{35}$$

Note that, for the continuous function defined on the bounded closed set, there must exist maximal and minimal solutions. Then for the optimization problem of (35), there must exist the maximal bounded $\varepsilon_1^*, \varepsilon_2^*, \ldots, \varepsilon_n^*$, $e_1^*, e_2^*, \ldots, e_m^*$ for $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n$, $e_1, e_2, \ldots, e_m$ respectively. Applying the function *fmincon* of Matlab™, we can get the numerical solution of $\varepsilon_1^*, \varepsilon_2^*, \ldots, \varepsilon_n^*$ and $e_1^*, e_2^*, \ldots, e_m^*$.

Theorem 2 *For Markov jump systems (1), under the condition of Assumption 1 and 2, if using the close loop regulator and the estimator (4) (9), then for any given real scalar $\gamma_i > 0$, $i \in S$ there exists a set Ω^* associated with a perturbation upper bound*

$$(\textstyle\sum_{s=1}^{n} \varepsilon_s W_s, \sum_{t=1}^{m} e_t V_t)$$

on uncertain noise covariances, where

$$\Omega^* = \{(\Delta W, \Delta V) : 0 \le \Delta W \le \sum_{s=1}^{n} \varepsilon_s^* W_s,$$
$$0 \le \Delta V \le \sum_{t=1}^{m} e_t^* V_t\}$$

and ε_s^*, e_t^* are the solution of (35) such that the inequality

$$\Delta \mathbf{J}_{i_0}(u, \Delta W, \Delta V) \le \gamma_i$$

is guaranteed to hold for $(\Delta W, \Delta V) \in \Omega^*$.

4. MINIMAX ROBUST JLQG REGULATOR

Setting U is all the admissible control set of (4), and

$$(\Delta W^*, \Delta V^*) = (\sum_{s=1}^{n} \varepsilon_s^* W_s, \sum_{t=1}^{m} e_t^* V_t)$$

$$(W^*, V^*) = (W + \Delta W^*, V + \Delta V^*)$$

It is reasonable choice for us to take the worst-case noise covariance (W^*, V^*) to design the Kalman filter

$$\dot{\hat{x}}^*(t) = A_i \hat{x}^*(t) + H_i^*[y(t) - C_i \hat{x}^*(t)] \tag{36}$$

$$H_i^* = P_{2i}^* C_i^T V^{*-1} \tag{37}$$

$$0 = A_i P_{2i}^* + P_{2i}^* A_i^T - P_{2i}^* C_i^T V^{*-1} C_i P_{2i}^* + \sum_{j=1}^{N} \pi_{ij} P_{2j}^* + W^* \tag{38}$$

So we can obtain the state feedback control laws with the state $\tilde{x}^*(t)$

$$u^*(t) = -G_i \hat{x}^*(t) \tag{39}$$

By the optimal estimation and control theory and **Lemma 1**, the following theorem comes into existence.

Theorem 3 *Under the condition of Theorem 2, Selecting the maximal matrix pair $(\Delta W^*, \Delta V^*)$ in Ω^* and applying the Kalman filters and control laws (36) (39) to Markov jump systems (1) with uncertain noise, then there exists saddle point $(u^*, \Delta W^*, \Delta V^*)$ satisfy the following saddle point inequality*

$$\begin{aligned} \Delta \mathbf{J}_{i_0}(u^*, \Delta W, \Delta V) &\le \Delta \mathbf{J}_{i_0}(u^*, \Delta W^*, \Delta V^*) \\ &\le \Delta \mathbf{J}_{i_0}(u, \Delta W^*, \Delta V^*) \end{aligned} \tag{40}$$

By Game Theory, the necessary and sufficient condition for equation (40) is

$$\min_{u\in U}\max_{(\Delta W,\Delta V)\in\Omega^*}\Delta\mathbf{J}_{i_0}(u,\Delta W,\Delta V)$$
$$=\max_{(\Delta W,\Delta V)\in\Omega^*}\min_{u\in U}\Delta\mathbf{J}_{i_0}(u,\Delta W,\Delta V)$$

that is, the JLQG optimal regulator under the worst-case condition is minmax robust JLQG regulator.

5. NUMERICAL EXAMPLE

Example 1: For the purpose of illustrating the developed theory, we consider a two-mode continuous-time Markov jump systems of the type (1).

$$A_1=\begin{bmatrix}-0.5 & 1.0\\ 0.3 & -2.5\end{bmatrix}, B_1=\begin{bmatrix}1.0 & 0\\ 0 & 1.0\end{bmatrix}, C_1=\begin{bmatrix}1.0 & 0\\ 0 & 1.0\end{bmatrix}$$
$$A_2=\begin{bmatrix}-1.0 & 0.1\\ 0.2 & -2.0\end{bmatrix}, B_2=\begin{bmatrix}1.0 & 0\\ 0 & 1.0\end{bmatrix}, C_2=\begin{bmatrix}1.0 & 0\\ 0 & 1.0\end{bmatrix}$$
$$\Pi=\begin{bmatrix}-2.0 & 2.0\\ 3.0 & -3.0\end{bmatrix}, W=\begin{bmatrix}1.0 & 0\\ 0 & 1.0\end{bmatrix}, V=\begin{bmatrix}1.0 & 0\\ 0 & 1.0\end{bmatrix}$$
$$Q_1=Q_2=\begin{bmatrix}2.0 & 0\\ 0 & 1.0\end{bmatrix},\quad R_1=R_2=\begin{bmatrix}1.0 & 0\\ 0 & 1.0\end{bmatrix}$$

Algorithm:

Step 1. Solve the couple Riccati equation (5) and (9), we obtain

$$P_1=\begin{bmatrix}0.9499 & 0.1986\\ 0.1986 & 0.2547\end{bmatrix}, P_2=\begin{bmatrix}0.8362 & 0.1029\\ 0.1029 & 0.2449\end{bmatrix}$$
$$P_{21}=\begin{bmatrix}0.5775 & 0.1377\\ 0.1377 & 0.2395\end{bmatrix}, P_{22}=\begin{bmatrix}0.5012 & 0.0758\\ 0.0758 & 0.2387\end{bmatrix}$$

Step 2. By (5) and (8), we have

$$G_1=\begin{bmatrix}0.9499 & 0.1986\\ 0.1986 & 0.2547\end{bmatrix}, G_2=\begin{bmatrix}0.8362 & 0.1029\\ 0.1029 & 0.2449\end{bmatrix}$$
$$H_1=\begin{bmatrix}0.5775 & 0.1377\\ 0.1377 & 0.2395\end{bmatrix}, H_2=\begin{bmatrix}0.5012 & 0.0758\\ 0.0758 & 0.2387\end{bmatrix}$$

Step 3. Solve the couple equation (23), we get

$$K_{13}=\begin{bmatrix}0.1869 & 0.0766\\ 0.0766 & 0.0391\end{bmatrix},\quad K_{23}=\begin{bmatrix}0.0814 & 0.0146\\ 0.0146 & 0.0139\end{bmatrix}$$

Step 4. Setting $\gamma_1 = \gamma_2 = 0.3$, and under the condition of **Theorem 2**, applying the function *fmincon* of Matlab™ to (35), we can get the perturbation upper bound

$$\varepsilon_1^*=0.0849,\quad \varepsilon_2^*=0.2441$$
$$e_1^*=0.4031,\quad e_2^*=1.9182$$

Step 5. Solve the couple equation (38), and calculate (37), to get

$$H_1^*=\begin{bmatrix}0.3586 & 0.3846\\ 0.0808 & 0.6730\end{bmatrix},\quad H_2^*=\begin{bmatrix}0.3106 & 0.2057\\ 0.0432 & 0.6699\end{bmatrix}$$

Step 6. Substituting H_1* and H_2* into (25), we obtain

$$K_{13}^*=\begin{bmatrix}0.1052 & 0.0376\\ 0.0376 & 0.0201\end{bmatrix},\quad K_{23}^*=\begin{bmatrix}0.0547 & 0.0113\\ 0.0113 & 0.0103\end{bmatrix}$$

Step 7. Applying the minmax robust JLQG regulator, we obtain the deviation of the control performance index under the worst-case condition:

when $i = 1$

$$\Delta J_{i_0}(u^*,\Delta W^*,\Delta V^*)=0.2225<0.3$$

for $i = 2$

$$\Delta J_{i_0}(u^*,\Delta W^*,\Delta V^*)=0.1190<0.3$$

Example 2: Consider a two-mode continuous-time Markov jump systems of the type (1).

$$A_1=\begin{bmatrix}1.0 & -1.2\\ 0.5 & 1.5\end{bmatrix}, B_1=\begin{bmatrix}1.0 & 0\\ 0 & 1.0\end{bmatrix}, C_1=\begin{bmatrix}1.0 & 0\\ 0 & 1.0\end{bmatrix}$$
$$A_2=\begin{bmatrix}-1.5 & 1.0\\ 0.5 & 2.0\end{bmatrix}, B_2=\begin{bmatrix}1.0 & 0\\ 0 & 1.0\end{bmatrix}, C_2=\begin{bmatrix}1.0 & 0\\ 0 & 1.0\end{bmatrix}$$
$$\Pi=\begin{bmatrix}-5.0 & 5.0\\ 10.0 & -10.0\end{bmatrix}, W=\begin{bmatrix}1.0 & 0\\ 0 & 2.0\end{bmatrix}, V=\begin{bmatrix}1.0 & 0\\ 0 & 1.0\end{bmatrix}$$
$$Q_1=Q_2=\begin{bmatrix}1.0 & 0\\ 0 & 1.0\end{bmatrix},\quad R_1=R_2=\begin{bmatrix}1.0 & 0\\ 0 & 1.0\end{bmatrix}$$

Using the Algorithm Step1-Step 3 in Example 1, we have

$$G_1=\begin{bmatrix}1.5584 & 0.2284\\ 0.2284 & 3.4839\end{bmatrix}, G_2=\begin{bmatrix}1.1855 & 0.3701\\ 0.3701 & 3.7411\end{bmatrix}$$
$$H_1=\begin{bmatrix}1.5618 & 0.2447\\ 0.2447 & 3.7217\end{bmatrix}, H_2=\begin{bmatrix}1.1879 & 0.3835\\ 0.3835 & 3.9884\end{bmatrix}$$

$$K_{13}=\begin{bmatrix}0.2347 & 0.1043\\ 0.1043 & 0.0863\end{bmatrix}, K_{23}=\begin{bmatrix}0.1052 & 0.0332\\ 0.0332 & 0.0295\end{bmatrix}$$

Setting $\gamma_1 = 0.2$, $\gamma_2 = 0.3$, then

$$\varepsilon_1^* = 0.0663, \quad \varepsilon_2^* = 0.1043$$
$$e_1^* = 0.2379, \quad e_2^* = 0.5654$$

$$H_1^*=\begin{bmatrix}1.0556 & 0.2334\\ 0.1005 & 0.3698\end{bmatrix}, \quad H_2^*=\begin{bmatrix}0.9432 & 0.2257\\ 0.0199 & 0.3797\end{bmatrix}$$
$$K_{13}^*=\begin{bmatrix}0.1355 & 0.0248\\ 0.0248 & 0.0196\end{bmatrix}, \quad K_{23}^*=\begin{bmatrix}0.0367 & 0.0253\\ 0.0253 & 0.0096\end{bmatrix}$$

when $i = 1$

$$\Delta J_{i_0}(u^*, \Delta W^*, \Delta V^*) = 0.1955 < 0.2$$

for $i = 2$

$$\Delta J_{i_0}(u^*, \Delta W^*, \Delta V^*) = 0.2768 < 0.3$$

The examples depict that applying the minmax robust JLQG regulator not only can minimize the worst quadratic object performance function of the uncertainty, but also can guaranteed the control performance index to be within a given freedom.

6. CONCLUSIONS

In the paper, the robust JLQG regulator of guaranteed control performance for continuous-time Markov jump systems with uncertain noise is considered. A simple and direct way is given to derive a convex set associated with a perturbation upper bound on uncertain noise covariances so that the deviation of the control object error performance index is guaranteed to be within the precision prescribed in actual problems. Furthermore, the worst performance yielded by the noise uncertainty can be minimized by a minimax robust regulator. The numerical examples have been used to illustrate the application of the proposed method.

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Biographies

Xiaobo Xiao received the B.S. and M.E. degree in control engineering from Air Force Engineering University, Xi'an, China, in 1997 and 2003. He is currently a Ph.D. candidate of Department of Automation at University of Science and Technology of China. His interesting is the estimation and control theory of Hybrid systems.

Hongsheng Xi received the M.S. degree in applying mathematics from University of Science and Technology of China, Hefei, China in 1977. He is currently Professor of Department of Automation at University of Science and Technology of China. His interesting is DEDS and others.

Jin Zhu received the B.S. in control engineering from University of Science and Technology of China, Hefei, China, in 2001. He is currently a Ph.D. candidate of Department of Automation at University of Science and Technology of China. His interesting is the control theory of Hybrid systems and nonlinear systems.

Haibo Ji received the B.S. degree in mechanics and mechanical engineering from Zhejiang University, Hangzhou, China, in 1984. He received the Ph.D. degree in mechanics and mechanical engineering from Peking University, Beijing, China, in 1990. He is currently Professor of Department of Automation at University of Science and Technology of China. His interesting is the control theory of nonlinear systems and others.

RESILIENT ADAPTIVE STABILIZATION OF UNCERTAIN TIME-DELAY SYSTEMS

M. S. Mahmoud [1,*], E. K. Boukas [2], A. Ismail [3]

[1] Chairman of the C-Group Technical Consultants, Orman-Dokki, Giza, Egypt

[2] Mechanical Engineering Department, ´Ecole Polytechnique de Montreal, Canada

[3] College of Engineering, UAE University, Al-Ain, United Arab Emirates

ABSTRACT

Delay-independent and delay-dependent robust stabilization schemes are established for a class of continuous-time systems with state-delays and norm-bounded uncertainties against controller gain variations. State-feedback resilient adaptive controllers are constructed for the case of known gain perturbation bounds and then extended to accommodate unknown norm-bounded perturbations. All the developed results are conveniently expressed in linear matrix inequalities (LMIs) format. Simulation results are presented to demonstrate the developed theory.

Keywords

Uncertain Systems, Adaptive Control, Time-Delay Systems, Robust Stabilization, LMIs.

1. INTRODUCTION

Considerable discussions on delays and their stabilization/destabilization effects in control systems have attracted the interests of numerous investigators in recent years [1-9] and it becomes quite clear that there are various sources for delays including finite capabilities of information processing among different parts of the system, inherent phenomena like mass transport flow and recycling and/or by product of computational delays.

Another research direction arises in the course of controller implementation based on different control design methods (including weighted H_2, H_∞, μ and l_1 synthesis techniques), it turns out that the controllers are very sensitive with respect to errors in the controller coefficients [5]. The sources for this include, but not limited to, imprecision in analogue-digital conversion, fixed word length, finite resolution instrumentation and numerical roundoff errors. By means of several examples, it is demonstrated [5] that relatively small perturbations in controller parameters could even destabilize the closed loop system. Hence, it is considered beneficial that the designed (nominal) controllers should be capable of tolerating some level of controller gain variations. This illuminates the controller fragility problem for which some relevant results are available in [1, 4] and further effort to alleviate this problem can also be found in [9, 12, 13].

The objective of this paper is to contribute to the further development of robust feedback stabilization for a class of linear state-delay systems against norm-bounded uncertainties and gain perturbations. In the present work, we focus on the design of delay-independent and delay-dependent adaptive controllers which guarantee the asymptotic stability of the closed-loop system for all admissible perturbations and uncertainties. Previous related work on adaptive stabilization of uncertain systems can be found in [6, 7, 8, 10] where the purpose of adaptation was to accommodate the bounds on uncertainty set. Here, we extend these methods to establish conditions for state-feedback delay-dependent stabilization of uncertain state-delay systems. These conditions are conveniently expressed in the form of linear matrix inequalities (LMIs). Numerical examples are provided to illustrate the theoretical developments.

Notations and Facts: In the sequel, the superscript "t" stands for matrix transposition, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space equipped with norm $||.||$, $\mathbb{R}^{n\times n}$ is the set of all $n \times m$ real matrices and the notation $W > 0$ for $W \in \mathbb{R}^{n\times n}$ means that W is symmetric and positive-definite. We use W^{-1} to denote the inverse of any square matrix W. The Lebsegue space $L_2[0,\infty)$ consists of square-integrable functions over the interval $[0,\infty)$. The symbol • will be used in some matrix expressions to induce a symmetric structure, that is if given matrices $L = L^t$ and $R = R^t$ of appropriate dimensions, then

$$\begin{bmatrix} L & \bullet \\ N & R \end{bmatrix} = \begin{bmatrix} L & N^t \\ N & R \end{bmatrix}$$

Fact 1: For any real matrices Σ_1, Σ_2 and Σ_3 with appropriate dimensions and $\Sigma_3^t\Sigma_3 \le I$, it follows that

$$\Sigma_1\Sigma_3\Sigma_2 + \Sigma_2^t\Sigma_3^t\Sigma_1^t \;\le\; \alpha\,\Sigma_1\Sigma_1^t + \alpha^{-1}\,\Sigma_2^t\Sigma_2, \quad \forall\alpha > 0$$

2. PROBLEM STATEMENT

We consider a plant P to be represented by the following class of time-delay systems:

$$\dot{x}(t) \;=\; A_\Delta x(t) + A_{\Delta d}x(t-\tau) + B_o u(t) \qquad (2.1)$$

where $x(t) \in \Re^n$ is the state vector; $u(t) \in \Re^p$ is the control input, τ is a constant-but-unknown time-delay and the uncertain matrices $A_\Delta \in \Re^{n\times n}$, $B_\Delta \in \Re^{n\times p}$ and $A_{\Delta d} \in \Re^{n\times n}$, are represented by

*Corresponding author: *E-mail:* magdim@yahoo.com

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$$[A_\Delta \quad A_{\Delta d}] = [A_o \quad A_d] + M\Delta_p(t)[N_a \quad N_d] \tag{2.2}$$

where $A_0 \in \Re^{n\times n}$, $B_0 \in \Re^{n\times p}$, $C_0 \in \Re^{q\times n}$, $A_d \in \Re^{n\times n}$, $M \in \Re^{n\times \alpha}$, $N_a \in \Re^{\beta\times n}$ and $N_d \in \Re^{\beta\times n}$, are real and known constant matrices with $\Delta_p(t)$ is a matrix of uncertainties and bounded in the form $\Delta_p(t)\Delta_p^t(t) \le I \forall t$. In the absence of uncertainties ($\Delta \equiv 0$), system (2.1) reduces to

$$\dot{x}(t) = A_o x(t) + A_d x(t-\tau) + B_o u(t) \tag{2.3}$$

It is a straightforward task to show that the nominal state-feedback controller

$$u(t) = 1/2\, B_o^t P\, x(t) \triangleq K_o\, x(t) \tag{2.4}$$

renders system (2.3) asymptotically stable for arbitrary constant delay $\tau \in [0 \rightarrow \tau^*]$ if given a matrix $0 < Q = Q^t \in \Re^{n\times n}$ there exists a matrix $0 < P = P^t \in \Re^{n\times n}$ such that the LMI

$$\begin{bmatrix} PA_o + A_o^t P & PA_d & PB_o \\ \bullet & -Q & 0 \\ \bullet & \bullet & -I \end{bmatrix} < 0 \tag{2.5}$$

has a feasible solution. In practical situations, there are at least two sources of inaccuracies when implementing controller (2.4). The first source is obviously due to the presence of uncertainties in the system matrices and the second source arises from gain perturbations due to various reasons [1, 4]. Therefore, it is naturally to consider, for a given nominal feedback controller $u(t) = K_0\, x(t)$, that the actual implemented controller is assumed to have two-terms:

$$u(t) = [K_o + \Delta K]\, x(t) + a(x) \tag{2.6}$$

where K_0 is the gain matrix to be determined, $K(t)$ represents the gain perturbation, which is assumed to be norm-bounded of the form:

$$||\Delta K(t)|| \le \beta \tag{2.7}$$

where $\beta > 0$ is an upper bound to be dealt with in the subsequent analysis and $a(x)$ is an adjustable state-dependent signal to be constructed to counteract the gain perturbation.

The problem of interest in this paper is to develop a feedback stabilization scheme that ensures that the closed-loop system of (2.1)-(2.3) is asymptotically stable. Among the various possible approaches, we aim at constructing an adaptive scheme to achieve the cited design objective. Needless to stress the salient features of adaptive stabilization methods are well-established [11].

The main results of this paper are divided into two parts: the first part deals with delay-independent stabilization and the second part treats delay-dependent stabilization. In either part and to achieve our goal, we will proceed in two stages. In the first stage, we attempt to construct an adaptive schemes for the uncertain time-delay system (2.1) assuming that the gain perturbation bound is known. Then in the second stage, we extend the results to accommodate bounded-but-known gain perturbations. All the established conditions are conveniently expressed in the form of linear matrix inequalities (LMIs)[3].

3. DELAY-INDEPENDENT STABILIZATION

3.1 Known Perturbation Bound

When the gain perturbation bound is known, then the purpose of adaptation is to accommodate the uncertainties of system (2.1). The following adaptive scheme is proposed

$$\begin{aligned} u(t) &= [K_o + \Delta K(t)]x(t) + \tilde{\mu}\, K_o\, e_n, \\ \dot{\tilde{\mu}} &= -g\,\tilde{\mu} + x^t K_o^t \tilde{\mu}^{-1} K_o x \;,\; \tilde{\mu}(0) = \mu^+ \;,\; g > 0 \end{aligned} \tag{3.1}$$

where K_o represents a control gain matrix to be determined in the sequel and $g > 0$, e_n denote a growth factor and a unit vector of order n. A convenient Lyapunov functional $V(.)$ is given by

$$V_n(t) = x^t(t)Px(t) + \int_{t-\tau}^{t} x^t(s)Qx(s)ds + 1/2\,\tilde{\mu}^2, \tag{3.2}$$

where $0 < P = P^t \in \mathbb{R}^{n\times n}$ and $0 < Q = Q^t \in \mathbb{R}^{n\times n}$. The following theorem summarizes the first main result:

Theorem 3.1 *System (2.1) under the adaptive controller (3.1) is asymptotically stable if for a given scalar $\beta > 0$, there exist matrices $0 < X = X^t \in \mathbb{R}^{n\times n}$, $0 < Q = Q^t \in \mathbb{R}^{n\times n}$, $Y \in \mathbb{R}^{m\times n}$, $0 < Z = Z^t \in \mathbb{R}^{n\times n}$, $\omega \in \mathbb{R}^{n\times 1}$ and scalars $g > 0$, $\varepsilon > 0$ satisfying the LMI*

$$\begin{bmatrix} \begin{matrix} A_oX + XA_o^t + B_oY + Y^tB_o^t \\ +Z + \beta L + \beta L^t + \varepsilon MM^t \end{matrix} & A_d & \omega & XN_a^t & Y^t \\ \bullet & -Q & 0 & 0 & 0 \\ \bullet & \bullet & -g & 0 & 0 \\ \bullet & \bullet & \bullet & -\varepsilon I & 0 \\ \bullet & \bullet & \bullet & \bullet & -I \end{bmatrix} < 0 \tag{3.3}$$

Moreover, the feedback gain is $K_o = YX^{-1}$.

Proof: Evaluation of the derivative of $V_n(t)$ along the solutions of system (2.1)-(2.2) using adaptive controller (3.1) with some algebraic manipulations yields (3.4) and (3.5).

From Lyapunov theory, it follows that $\dot{V}_n(t) < 0$ is guaranteed if $\Xi_n < 0$. By [9] with **Fact 1**, it follows that, see (3.6)

Using the congruence transformation $T = diag[X\ I\ I]$, $X = P^{-1}$, defining $Y = K_oX$ and invoking the linearizations $Z = XQX$, $L = B_o I_{mn} X$, $\omega = B_o K_o e_n$, it is a straightforward task to show that the stability condition holds if there exist a scaler $\varepsilon > 0$ such that, see (3.7):

A Schur complement will convert (3.7) to (3.3) and thus the proof is completed.

Remark 3.1 *The dynamical relation of μ consists of two-terms: one is growth factor and the other is a product of μ and x so as to preserve inter-coupling between the states and the gain factor. The value of the growth factor $g > 0$ guarantees the asymptotic stability of system (2.1) under controller (3.1) and its magnitude gives an indication of the speed of response.*

$$
\begin{aligned}
\dot{V}_n(t) &= x^t(t)\Big[PA_{\Delta c} + A_{\Delta c}^t P + Q + K_o^t K_o + \beta P B_o I_{mn} + \beta I_{nm} B_o^t P\Big] x(t) + 2x^t P A_{\Delta d} x(t-\tau) \\
&- x^t(t-\tau)\, Q\, x(t-\tau) - g\,\tilde{\mu}^2 \; + 2x^t P B_o K_o e_n \tilde{\mu} \\
&= \begin{bmatrix} x \\ x(t-\tau) \\ \tilde{\mu} \end{bmatrix}^t \begin{bmatrix} \begin{array}{c} PA_{\Delta c} + A_{\Delta c}^t P + Q \\ +K_o^t K_o + \beta P B_o I_{mn} + \beta I_{nm} B_o^t P \end{array} & PA_{\Delta d} & PB_o K_o e_n \\ \bullet & -Q & 0 \\ \bullet & \bullet & -g \end{bmatrix} \begin{bmatrix} x \\ x(t-\tau) \\ \tilde{\mu} \end{bmatrix} \\
&= \begin{bmatrix} x \\ x(t-\tau) \\ \tilde{\mu} \end{bmatrix}^t \Xi_n \begin{bmatrix} x \\ x(t-\tau) \\ \tilde{\mu} \end{bmatrix}
\end{aligned}
\tag{3.4}
$$

$$
\begin{aligned}
A_{\Delta c} &= A_\Delta + B_o K_o \\
&= [A_o + B_o K_o] + \Delta A = A_c + \Delta A
\end{aligned}
\tag{3.5}
$$

$$
\begin{aligned}
& \begin{bmatrix} \begin{array}{c} PA_{\Delta c} + A_{\Delta c}^t P + Q \\ +K_o^t K_o + \beta P B_o I_{mn} + \beta I_{nm} B_o^t P \end{array} & PA_{\Delta d} & PB_o K_o e_n \\ \bullet & -Q & 0 \\ \bullet & \bullet & -g \end{bmatrix} \\
\Xi_n \;\; &= \begin{bmatrix} \begin{array}{c} PA_c + A_c^t P + Q \\ +K_o^t K_o + \beta P B_o I_{mn} + \beta I_{nm} B_o^t P \end{array} & PA_d & PB_o K_o e_n \\ \bullet & -Q & 0 \\ \bullet & \bullet & -g \end{bmatrix} \\
&+ \begin{bmatrix} P\Delta A + \Delta A^t P & P\Delta A_d & 0\,\text{(8)} \\ \bullet & 0 & 0 \\ \bullet & \bullet & 0 \end{bmatrix} \\
&\le \begin{bmatrix} \begin{array}{c} PA_{\Delta c} + A_{\Delta c}^t P + Q\,\text{(9)} \\ +K_o^t K_o + \beta P B_o I_{mn} + \beta I_{nm} B_o^t P \end{array} & PA_{\Delta d} & PB_o K_o e_n \\ \bullet & -Q & 0 \\ \bullet & \bullet & -g \end{bmatrix} \\
&+ \;\varepsilon \begin{bmatrix} PM \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} PM \\ 0 \\ 0 \end{bmatrix}^t + \varepsilon^{-1} \begin{bmatrix} N_a \\ N_d \\ 0 \end{bmatrix} \begin{bmatrix} N_a \\ N_d \\ 0 \end{bmatrix}^t \\
&= \begin{bmatrix} \begin{array}{c} PA_c + A_c^t P + Q \\ +K_o^t K_o + \beta P B_o I_{mn} + \beta I_{nm} B_o^t P \\ +\varepsilon P M M^t P + \varepsilon^{-1} N_a^t N_a \end{array} & PA_d + \varepsilon^{-1} N_a^t N_d & PB_o K_o e_n \\ \bullet & -Q + \varepsilon^{-1} N_d^t N_d & 0 \\ \bullet & \bullet & -g \end{bmatrix} < 0
\end{aligned}
\tag{3.6}
$$

$$
\begin{bmatrix} \begin{array}{c} A_c X + X A_c^t + Z \\ +Y^t Y + \beta L + \beta L^t \\ +\varepsilon M M^t + \varepsilon^{-1} X N_a^t N_a X \end{array} & A_d + \varepsilon^{-1} X N_a^t N_d & B_o K_o e_n \\ \bullet & -Q + \varepsilon^{-1} N_d^t N_d & 0 \\ \bullet & \bullet & -g \end{bmatrix} < 0
\tag{3.7}
$$

3.2 Unknown Gain Perturbation Bound

Now we consider the application of controller (2.6) subject to bound (2.7) where β is unknown. The following adaptive scheme is then proposed

$$\begin{aligned} u(t) &= K_o\, x(t) + \bar{\mu}\, K_o\, e_n + \beta\, K_o\, e_n, \\ \dot{\bar{\mu}} &= -g\,\bar{\mu} \;+\; x^t K_o^t \bar{\mu}^{-1} K_o x + \beta\, \|x\|^2 \;\;,\;\; \bar{\mu}(0) = \mu^* \\ \dot{\beta} &= -h\,\beta - \bar{\mu}\,\|x\|^2 \;+\; \beta^{-1} x^t R x, \quad \beta(0) = \beta^*, \;\; R \in \mathbb{R}^{n\times n} \end{aligned} \tag{3.8}$$

where $h > 0$ represents a growth factor. Note that scheme (3.8) is constructed in the same way as scheme (3.1). A convenient Lyapunov functional $V(.)$ is given by

$$V_b(t) = x^t(t)Px(t) + \int_{t-\tau}^{t} x^t(s)Qx(s)ds + 1/2\,\bar{\mu}^2 + 1/2\,\beta^2 \tag{3.9}$$

The following theorem summarizes the second main result:

Theorem 3.2 *System (2.1) under the adaptive controller (3.8) is asymptotically stable if there exist matrices* $0 < X = X^t \in \mathbb{R}^{n\times n}$, $0 < Q = Q^t \in \mathbb{R}^{n\times n}$, $Y \in \mathbb{R}^{m\times n}$, $0 < Z = Z^t \in \mathbb{R}^{n\times n}$, $0 < W = W^t \in \mathbb{R}^{n\times n}$, $\omega \in \mathbb{R}^{n\times 1}$ *and scalars* $g > 0$, $h > 0$, $\varepsilon > 0$ *satisfying the LMI*

$$\begin{bmatrix} \begin{array}{c} A_oX + XA_o^t + B_oY + Y^tB_o^t \\ +Z + W + \varepsilon MM^t \end{array} & A_d & \omega & \omega & XN_a^t & Y^t \\ \bullet & -Q & 0 & 0 & 0 & 0 \\ \bullet & \bullet & -g & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & -h & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & -\varepsilon I & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & -I \end{bmatrix} < 0 \tag{3.10}$$

Moreover, the feedback gain is $K = YX^{-1}$.

Proof: An evaluation of the derivative of $V_b(t)$ along the solutions of system (2.1)-(2.2) using (3.5) and adaptive controller (3.1), yields:

$$\begin{aligned} \dot{V}_b(t) &= x^t(t)\Big[PA_{\Delta c} + A_{\Delta c}^tP + Q + R + K_o^tK_o\Big]x(t) + 2x^tPA_{\Delta d}x(t-\tau) \\ &- x^t(t-\tau)\,Q\,x(t-\tau) - g\,\tilde{\mu}^2 + 2x^tPB_oK_oe_n - h\,\beta^2 + 2x^tPB_oK_oe_n\beta \\ &= \eta^t \begin{bmatrix} \begin{array}{c} PA_{\Delta c} + A_{\Delta c}^tP + Q \\ +K_o^tK_o + R \end{array} & PA_{\Delta d} & PB_oK_oe_n & PB_oK_oe_n \\ \bullet & -Q & 0 & 0 \\ \bullet & \bullet & -g & 0 \\ \bullet & \bullet & \bullet & -h \end{bmatrix} \eta \\ &= \eta^t\, \Xi_f\, \eta, \quad \eta = \begin{bmatrix} x^t & x^t(t-\tau) & \tilde{\mu}^t & \beta^t \end{bmatrix}^t \end{aligned} \tag{3.11}$$

Following parallel development to **Theorem (3.2)**, it is readily evident that the stability condition $\dot{V}_n(t) < 0$ holds if there exist scalers $\varepsilon > 0$

$$\begin{bmatrix} \begin{array}{c} PA_c + A_c^tP + Q \\ +K_o^tK_o + R \\ +\varepsilon PMM^tP + \varepsilon^{-1}N_a^tN_a \end{array} & PA_d + \varepsilon^{-1}N_a^tN_d & PB_oK_oe_n & PB_oK_oe_n \\ \bullet & -Q + \varepsilon^{-1}N_d^tN_d & 0 & 0 \\ \bullet & \bullet & -g & 0 \\ \bullet & \bullet & \bullet & -h \end{bmatrix} < 0 \tag{3.12}$$

Using the congruence transformation $T = diag[X\, I\, I\, I]$, $X = P^{-1}$ and defining $Y = K_oX$ and invoking the linearizations $Z = XQX$, $W = XRX$, $\omega = B_oK_oe_n$, it follows that Schur operations converts (3.12) to (3.10) and hence the proof is completed.

Remark 3.2 *In a similar way, each of the dynamical relations of* $\dot{\mu}$ *and consists of three-terms: one is growth factor and the others are pair-wise products of* μ, *and x so as to preserve inter-coupling between the states and the gain factors. The values of the growth factor g > 0, h > 0 guarantee the asymptotic stability of the closed-loop system and their magnitudes give indication of the speed of response.*

4. DELAY-DEPENDENT STABILIZATION

In order to develop delay-dependent criteria for stabilization, the dynamic model (2.1) is represented by the descriptor form:

$$\begin{aligned} \dot{x}(t) &= \sigma(t) \\ 0 &= -\sigma(t) + (A_\Delta + A_{\Delta d})x(t) - A_{\Delta d} \int_{t-\tau}^{t} \sigma(s)ds + B_o u(t) \end{aligned} \tag{4.1}$$

Initially, we focus on the nominal feedback stabilization

4.1 Nominal State Feedback Stabilization

Under the control law $u(t) = K_o x(t)$ where K_o is the unknown gain matrix, system (4.1) using (2.2) becomes.

$$\begin{aligned} \dot{x}(t) &= \sigma(t) \\ 0 &= -\sigma(t) + A_{\Delta K d}x(t) - A_{\Delta d} \int_{t-\tau}^{t} \sigma(s)ds \end{aligned} \tag{4.2}$$

$$\begin{aligned} A_{\Delta K d} &= (A_\Delta + A_{\Delta d}) + B_o K_o = A_{od} + B_o K_o + M\Delta_p(t)N_{ad} \\ A_{od} &= A_o + A_d, \quad N_{ad} = N_a + N_d \end{aligned} \tag{4.3}$$

We introduce, for convenience, the following matrices

$$\bar{P} = \mathsf{U}\,\mathbb{P}\ ;\ \mathsf{U} = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}\ ;\ \mathbb{P} = \begin{bmatrix} P_x & 0 \\ P_d & P_\sigma \end{bmatrix}$$

$$E_1 = \begin{bmatrix} I \\ 0 \end{bmatrix},\ E_2 = \begin{bmatrix} 0 \\ I \end{bmatrix},\ Y = \mathbb{P}^{-1} = \begin{bmatrix} Y_x & 0 \\ Y_d & Y_\sigma \end{bmatrix} \tag{4.4}$$

$$\mathcal{A}_{\Delta K d} = \begin{bmatrix} 0 & I \\ A_{\Delta K d} & -I \end{bmatrix},\ \bar{A}_{\Delta d} = \begin{bmatrix} 0 \\ A_{\Delta d} \end{bmatrix} \tag{4.5}$$

and provide the following stability result.

Theorem 4.1 *System (4.2) is delay-dependent stabilizable under the controller $u(t) = K_o x(t)$ if there exist matrices $0 < Y_x^t = Y_x \in \mathbb{R}^{n \times n}$, $Y_\sigma \in \mathbb{R}^{n \times n}$, $Y_d \in \mathbb{R}^{n \times n}$, $0 < Q_\sigma^t = Q_\sigma \in \mathbb{R}^{n \times n}$, $Z \in \mathbb{R}^{n \times n}$ and scalars $\varepsilon > 0$ satisfying the following LMI*

$$\begin{bmatrix} Y_d + Y_d^t & Y_x A_{od}^t + Z^t B_o^t - Y_d^t & 0 & Y_x N_{ad}^t & \tau Y_d^t Q_\sigma \\ \bullet & -Y_\sigma - Y_\sigma^t + \varepsilon M M^t & \tau A_d & 0 & \tau Y_\sigma^t Q_\sigma \\ \bullet & \bullet & -\tau Q_\sigma & \tau N_d^t & 0 \\ \bullet & \bullet & \bullet & -\varepsilon I & 0 \\ \bullet & \bullet & \bullet & \bullet & -\tau Q_\sigma \end{bmatrix} < 0 \tag{4.6}$$

Moreover, the gain matrix is $K_0 = Z Y_x^{-1}$.

Proof: Let the Lyapunov-Krasovskii functional $V(\cdot)$ of the transformed system be selected as:

$$\begin{aligned} V_t(t) &\triangleq V_x(t) + V_\sigma(t) \\ V_x(t) &= x^t(t) P_x x(t)\ ,\ V_\sigma(t) = \int_{-\tau}^{0} \int_{t+\theta}^{t} \sigma^t(s)\, Q_\sigma\, \sigma(s)\, ds\, d\,\theta \\ & \quad 0 < P_x^t = P_x \in \mathbb{R}^{n\times n},\ \ 0 < Q_\sigma^t = Q_\sigma \in \mathbb{R}^{n\times n} \end{aligned} \tag{4.7}$$

Using (4.2) and (4.7), we get:

$$\begin{aligned} \dot{V}_x(t) &= 2x^t(t) P_x^t\, \dot{x}(t) = 2[x^t(t) \quad \sigma^t(t)]\mathbb{P}^t[\dot{x}^t(t) \quad 0]^t \\ &= 2[x^t(t) \quad \sigma^t(t)]\, \mathbb{P}^t \begin{bmatrix} \sigma(t) \\ -\sigma(t) + A_{\Delta K d}x(t) \end{bmatrix} \\ &+ 2[x^t(t) \quad \sigma^t(t)]\, \mathbb{P}^t \begin{bmatrix} 0 \\ -A_{\Delta d} \int_{t-\tau}^{t} \sigma(s)ds \end{bmatrix} \end{aligned} \tag{4.8}$$

Simple manipulations using (4.2) yield:

$$2[x^t(t) \quad \sigma^t(t)]\, \mathbb{P}^t \begin{bmatrix} \sigma(t) \\ -\sigma(t) + A_{\Delta K d} x(t) \end{bmatrix} = [x^t(t) \quad \sigma^t(t)] \left\{ \mathbb{P}^t \mathcal{A}_{\Delta K d} + \mathcal{A}_{\Delta K d}^t \mathbb{P} \right\} \begin{bmatrix} x(t) \\ \sigma(t) \end{bmatrix} \tag{4.9}$$

Using **Fact 1**, it follows that

$$\begin{aligned} & 2[x^t(t) \quad \sigma^t(t)]\, \mathbb{P}^t \begin{bmatrix} 0 \\ -A_{\Delta d} \int_{t-\tau}^{t} \sigma(s) ds \end{bmatrix} \\ = \;& 2 \int_{t-\tau}^{t} [x^t(t) \quad \sigma^t(t)]\, \mathbb{P}^t \bar{A}_{\Delta d} \sigma(s) ds \\ \leq \;& \tau \begin{bmatrix} x(t) \\ \sigma(t) \end{bmatrix}^t \mathbb{P}^t \bar{A}_{\Delta d} Q_\sigma^{-1} \bar{A}_{\Delta d}^t \mathbb{P} \begin{bmatrix} x(t) \\ \sigma(t) \end{bmatrix} + \int_{t-\tau}^{t} \sigma^t(s) Q_\sigma \sigma(s)\, ds \end{aligned} \tag{4.10}$$

Therefore from (4.8)-(4.10) we get:

$$\begin{aligned} \dot{V}_x \;\leq\; & \begin{bmatrix} x(t) \\ \sigma(t) \end{bmatrix}^t \left\{ \mathbb{P}^t \mathcal{A}_{\Delta K d} + \mathcal{A}_{\Delta K d}^t \mathbb{P} + \tau\, \mathbb{P}^t \bar{A}_{\Delta d} Q_\sigma^{-1} \bar{A}_{\Delta d}^t \mathbb{P} \right\} \begin{bmatrix} x(t) \\ \sigma(t) \end{bmatrix} \\ + & \int_{t-\tau}^{t} \sigma^t(s) Q_\sigma \sigma(s)\, ds \end{aligned}$$

It is straightforward to show that, (4.11)

$$\dot{V}_\sigma = \tau\, \sigma^t(t) Q_\sigma \sigma(t) - \int_{-\tau}^{0} \sigma^t(t+\theta) Q_\sigma \sigma(t+\theta)\, d\theta$$

Using (4.4)-(4.5) and (4.11), it follows that

$$\begin{aligned} \dot{V}_t \;=\; & \dot{V}_x + \dot{V}_\sigma \\ =\; & \begin{bmatrix} x(t) \\ \sigma(t) \end{bmatrix}^t \left\{ \mathbb{P}^t \mathcal{A}_{\Delta K d} + \mathcal{A}_{\Delta K d}^t \mathbb{P} + \tau\, \mathbb{P}^t \bar{A}_{\Delta d} Q_\sigma^{-1} \bar{A}_{\Delta d}^t \mathbb{P} + \tau\, E_2 Q_\sigma E_2^t \right\} \begin{bmatrix} x(t) \\ \sigma(t) \end{bmatrix} \\ \triangleq\; & \zeta^t(t)\, \Pi\, \zeta(t), \quad \zeta(t) = [x^t(t) \quad \sigma^t(t)]^t \end{aligned} \tag{4.12}$$

Since $V_t(t) > 0$, it follows from Lyapunov theory that $\dot{V}_t > 0$, $\forall \zeta(t) \neq 0$ is a sufficient condition for stability which, in turn, entails that $\Pi < 0$. The latter condition under the congruence transformation $Y = \mathbb{P}^{-1}$ becomes:

(4.13)

$$\mathcal{A}_{\Delta K d} Y + Y^t \mathcal{A}_{\Delta K d}^t + \tau\, \bar{A}_{\Delta d} Q_\sigma^{-1} \bar{A}_{\Delta d}^t + \tau\, Y^t E_2 Q_\sigma E_2^t Y \;<\; 0$$

Algebraic manipulation of (4.14) using (4.5) and Schur complement yields

$$\begin{bmatrix} Y_d + Y_d^t + \tau Y_d^t Q_\sigma Y_d & Y_x A_{\Delta K d}^t - Y_d^t + \tau Y_d^t Q_\sigma Y_\sigma & 0 \\ \bullet & -Y_\sigma - Y_\sigma^t + \tau Y_\sigma^t Q_\sigma Y_\sigma & \tau A_{\Delta d} \\ \bullet & \bullet & -\tau Q_\sigma \end{bmatrix} \;<\; 0 \tag{4.14}$$

In terms of $Z = K_o Y_x$ and applying **Fact 1**, inequality (4.15) reduces to

(4.15)

$$\begin{bmatrix} Y_d + Y_d^t + \tau Y_d^t Q_\sigma Y_d & Y_x A_{od}^t + Z^t B_o^t - Y_d^t + \tau Y_d^t Q_\sigma Y_\sigma & 0 \\ \bullet & -Y_\sigma - Y_\sigma^t + \tau Y_\sigma^t Q_\sigma Y_\sigma & \tau A_d \\ \bullet & \bullet & -\tau Q_\sigma \end{bmatrix}$$
$$+ \quad \varepsilon \begin{bmatrix} 0 \\ M \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ M \\ 0 \end{bmatrix}^t + \varepsilon^{-1} \begin{bmatrix} Y_x N_{ad}^t \\ 0 \\ \tau N_d^t \end{bmatrix} \begin{bmatrix} Y_x N_{ad}^t \\ 0 \\ \tau N_d^t \end{bmatrix}^t < 0 \tag{4.16}$$

for some ε > 0. Further Schur complement operations bring inequality (4.16) to LMI (4.6) as desired.

Remark 4.1 *In implementing the LMI (4.6), we specify τ and check the feasibility. The process is continued till a feasible solution is failed to achieve and we record this point as τ^*. This means that the closed-loop system is delay-dependent asymptotically stable over the range $0 \le \tau \le \tau^*$.*

In the following sections, we consider adaptive stabilization schemes to deal with gain perturbations.

4.2 Adaptive Feedback Stabilization: Known Perturbation Bound

Consider that the gain matrix K_o is subject to perturbation K_o whose upper bound is known. In this case the adaptive scheme (3.1) is applied to the transformed system (4.1) to yield the closed-loop system

$$\dot{x}(t) = \sigma(t) \tag{4.17}$$
$$0 = -\sigma(t) + \hat{A}_{\Delta K d} x(t) + \tilde{\mu}\, B_o K_o\, e_n - A_{\Delta d} \int_{t-\tau}^{t} \sigma(s) ds$$

$$\begin{aligned} \hat{A}_{\Delta K d} &= (A_\Delta + A_{\Delta d}) + B_o K_o + \beta B_o I_{mn} \\ &= A_{od} + B_o K_o + \beta B_o I_{mn} + M \Delta_p(t) N_{ad} \end{aligned} \tag{4.18}$$

Next consider the Lyapunov-Krasovskii functional $V(\cdot)$ as follows:

$$\begin{aligned} V_{nt}(t) &\triangleq V_x(t) + V_\sigma(t) + 1/2\, \tilde{\mu}^2 \\ V_x(t) &= x^t(t) P_x x(t), \;\; V_\sigma(t) = \int_{-\tau}^{0} \int_{t+\theta}^{t} \sigma^t(s)\, Q_\sigma\, \sigma(s)\, ds\, d\theta \\ & \quad 0 < P_x^t = P_x \in \mathbb{R}^{n \times n}, \;\; 0 < Q_\sigma^t = Q_\sigma \in \mathbb{R}^{n \times n} \end{aligned} \tag{4.19}$$

Using (4.17) and (4.19), we get:

$$\begin{aligned} \dot{V}_x(t) &= 2x^t(t) P_x^t\, \dot{x}(t) \;=\; 2[x^t(t) \;\; \sigma^t(t)] \mathbb{P}^t [\dot{x}^t(t) \;\; 0]^t \\ &= 2[x^t(t) \;\; \sigma^t(t)]\, \mathbb{P}^t \begin{bmatrix} \sigma(t) \\ -\sigma(t) + \hat{A}_{\Delta K d} x(t) \end{bmatrix} \\ &+ 2[x^t(t) \;\; \sigma^t(t)]\, \mathbb{P}^t \begin{bmatrix} 0 \\ B_o K_o\, e_n \end{bmatrix} \tilde{\mu} \\ &+ 2[x^t(t) \;\; \sigma^t(t)]\, \mathbb{P}^t \begin{bmatrix} 0 \\ -A_{\Delta d} \int_{t-\tau}^{t} \sigma(s) ds \end{bmatrix} \end{aligned} \tag{4.20}$$

Simple manipulations using (4.17) yield:

$$2[x^t(t) \;\; \sigma^t(t)]\, \mathbb{P}^t \begin{bmatrix} \sigma(t) \\ -\sigma(t) + \hat{A}_{\Delta K d} x(t) \end{bmatrix} = \tag{4.21}$$
$$[x^t(t) \;\; \sigma^t(t)] \left\{ \mathbb{P}^t \hat{\mathcal{A}}_{\Delta K d} + \hat{\mathcal{A}}_{\Delta K d}^t \mathbb{P} \right\} \begin{bmatrix} x(t) \\ \sigma(t) \end{bmatrix}$$

$$\hat{\mathcal{A}}_{\Delta K d} = \begin{bmatrix} 0 & I \\ \hat{A}_{\Delta K d} & -I \end{bmatrix} \tag{4.22}$$

Using **Fact 1**, it follows that

$$\begin{aligned} & 2[x^t(t) \quad \sigma^t(t)]\, \mathbb{P}^t \begin{bmatrix} 0 \\ -A_{\Delta d} \int_{t-\tau}^{t} \sigma(s) ds \end{bmatrix} \\ = \; & 2 \int_{t-\tau}^{t} [x^t(t) \quad \sigma^t(t)]\, \mathbb{P}^t \bar{A}_{\Delta d} \sigma(s) ds \\ \leq \; & \tau \begin{bmatrix} x(t) \\ \sigma(t) \end{bmatrix}^t \mathbb{P}^t \bar{A}_{\Delta d} Q_\sigma^{-1} \bar{A}_{\Delta d}^t \mathbb{P} \begin{bmatrix} x(t) \\ \sigma(t) \end{bmatrix} + \int_{t-\tau}^{t} \sigma^t(s) Q_\sigma \sigma(s)\, ds \end{aligned} \tag{4.23}$$

Therefore from (4.20)-(4.23) we get:

$$\begin{aligned} \dot{V}_x \leq \; & \begin{bmatrix} x(t) \\ \sigma(t) \end{bmatrix}^t \left\{ \mathbb{P}^t \hat{A}_{\Delta K d} + \hat{A}_{\Delta K d}^t \mathbb{P} + \tau\, \mathbb{P}^t \bar{A}_{\Delta d} Q_\sigma^{-1} \bar{A}_{\Delta d}^t \mathbb{P} \right\} \begin{bmatrix} x(t) \\ \sigma(t) \end{bmatrix} \\ + \; & \int_{t-\tau}^{t} \sigma^t(s) Q_\sigma \sigma(s)\, ds \; + \; 2 \begin{bmatrix} x(t) \\ \sigma(t) \end{bmatrix}^t \mathbb{P}^t \begin{bmatrix} 0 \\ B_o K_o\, e_n \end{bmatrix} \tilde{\mu} \end{aligned} \tag{4.24}$$

Let

$$\bar{K}_o^t \bar{K}_o = \begin{bmatrix} K_o^t K_o & 0 \\ 0 & 0 \end{bmatrix}$$

Using (4.4)-(4.5), (4.12) and (4.24), it follows that

$$\begin{aligned} \dot{V}_{nt} = \; & \dot{V}_x + \dot{V}_\sigma + \tilde{\mu}\dot{\tilde{\mu}} \\ = \; & \begin{bmatrix} x(t) \\ \sigma(t) \\ \tilde{\mu} \end{bmatrix}^t \begin{bmatrix} \begin{matrix} \mathbb{P}^t \hat{A}_{\Delta K d} + \hat{A}_{\Delta K d}^t \mathbb{P} + \bar{K}_o^t \bar{K}_o + \\ \tau \mathbb{P}^t \bar{A}_{\Delta d} Q_\sigma^{-1} \bar{A}_{\Delta d}^t \mathbb{P} + \tau E_2 Q_\sigma E_2^t \end{matrix} & \cdot & \mathbb{P}^t \begin{bmatrix} 0 \\ B_o K_o\, e_n \end{bmatrix} \\ \bullet & \cdot & \cdot \\ \bullet & \bullet & -g \end{bmatrix} \begin{bmatrix} x(t) \\ \sigma(t) \\ \tilde{\mu} \end{bmatrix} \\ \triangleq \; & \psi^t(t)\, \Pi_n\, \psi(t), \quad \psi(t) = [x^t(t) \quad \sigma^t(t) \quad \tilde{\mu}]^t \end{aligned} \tag{4.25}$$

From Lyapunov theory that $\dot{V}_{nt} < 0$, $\forall \zeta(t) \neq 0$ is a sufficient condition for stability which, in turn, entails that $\Pi_n < 0$. The latter condition under the congruence transformation $[\mathbb{P}^{-1}\ I]$, $Y = \mathbb{P}^{-1}$ becomes:

$$\begin{bmatrix} \begin{matrix} \hat{A}_{\Delta K d} Y + Y^t \hat{A}_{\Delta K d}^t + Y^t \bar{K}_o^t \bar{K}_o Y + \\ \tau \bar{A}_{\Delta d} Q_\sigma^{-1} \bar{A}_{\Delta d}^t + \tau Y^t E_2 Q_\sigma E_2^t Y \end{matrix} & \cdot & \begin{bmatrix} 0 \\ B_o K_o\, e_n \end{bmatrix} \\ \cdot & \cdot & \cdot \\ \bullet & \bullet & -g \end{bmatrix} < 0 \tag{4.26}$$

The following stability result is established.

Theorem 4.2 *System (4.2) is delay-dependent stabilizable under the adaptive controller (3.1) given the bound β if there exist matrices $0 < Y_x^t = Y_x \in \mathbb{R}^{n \times n}$, $Y_\sigma \in \mathbb{R}^{n \times n}$, $Y_d \in \mathbb{R}^{n \times n}$, $0 < Q_\sigma^t = Q_\sigma \in \mathbb{R}^{n \times n}$, $Z \in \mathbb{R}^{n \times n}$ and a scalar $\varepsilon > 0$ satisfying the following LMI*

$$\begin{bmatrix} Y_d + Y_d^t & Y_x A_{od}^t + Z^t B_o^t + \beta Y_x B_o I_{mn} - Y_d^t & 0 & 0 & Y_x N_{ad}^t & \tau Y_d^t Q_\sigma & Z \\ \bullet & -Y_\sigma - Y_\sigma^t + \varepsilon M M^t & \tau A_d & \omega & 0 & \tau Y_\sigma^t Q_\sigma & 0 \\ \bullet & \bullet & -\tau Q_\sigma & 0 & \tau N_d^t & 0 & 0 \\ \bullet & \bullet & \bullet & -g & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & -\varepsilon I & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & -\tau Q_\sigma & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -I \end{bmatrix} < 0 \tag{4.27}$$

Moreover, the gain matrix is $K_0 = Z Y_x^{-1}$.

Proof: Follows from a parallel development to **Theorem 4.1** and using $\omega = B_o K_o e_n$.

4.3 Adaptive Feedback Stabilization: Unknown Perturbation Bound

When the upper bound β is unknown, the adaptive scheme (3.8) will be used applied to the transformed system (4.1) to yield the closed-loop system

$$\begin{aligned}\dot{x}(t) &= \sigma(t) \\ 0 &= -\sigma(t) + A_{\Delta Kd}x(t) + \bar{\mu}\, B_oK_o\, e_n + \beta\, B_oK_o\, e_n - A_{\Delta d}\int_{t-\tau}^{t} \sigma(s)ds\end{aligned} \tag{4.28}$$

where $A_{\Delta Kd}$ is given by (4.3). A convenient Lyapunov-Krasovskii functional is as follows:

$$\begin{aligned}V_{ut}(t) &\stackrel{\Delta}{=} V_x(t) + V_\sigma(t) + 1/2\,\bar{\mu}^2 + 1/2\,\beta^2 \\ V_x(t) &= x^t(t)P_x x(t),\ \ V_\sigma(t) = \int_{-\tau}^{0}\int_{t+\theta}^{t} \sigma^t(s)\, Q_\sigma\, \sigma(s)\, ds\, d\,\theta \\ & \quad 0 < P_x^t = P_x \in \mathbb{R}^{n\times n},\ \ 0 < Q_\sigma^t = Q_\sigma \in \mathbb{R}^{n\times n}\end{aligned} \tag{4.29}$$

Proceeding similar to the foregoing section, we get:

$$\begin{aligned}\dot{V}_x(t) &= 2x^t(t)P_x^t\,\dot{x}(t) = 2[x^t(t)\ \ \sigma^t(t)]\mathbb{P}^t[\dot{x}^t(t)\ \ 0]^t \\ &= 2[x^t(t)\ \ \sigma^t(t)]\,\mathbb{P}^t\begin{bmatrix}\sigma(t) \\ -\sigma(t) + A_{\Delta Kd}x(t)\end{bmatrix} \\ &+ 2[x^t(t)\ \ \sigma^t(t)]\,\mathbb{P}^t\begin{bmatrix}0 \\ B_oK_o\, e_n\end{bmatrix}\bar{\mu} \\ &+ 2[x^t(t)\ \ \sigma^t(t)]\,\mathbb{P}^t\begin{bmatrix}0 \\ B_oK_o\, e_n\end{bmatrix}\beta \\ &+ 2[x^t(t)\ \ \sigma^t(t)]\,\mathbb{P}^t\begin{bmatrix}0 \\ -A_{\Delta d}\int_{t-\tau}^{t}\sigma(s)ds\end{bmatrix}\end{aligned} \tag{4.30}$$

It follows from (4.9)-(4.10) and (4.30) that

$$\begin{aligned}\dot{V}_x &\le \begin{bmatrix}x(t) \\ \sigma(t)\end{bmatrix}^t\left\{\mathbb{P}^t\mathcal{A}_{\Delta Kd} + \mathcal{A}_{\Delta Kd}^t\mathbb{P} + \tau\,\mathbb{P}^t\bar{A}_{\Delta d}Q_\sigma^{-1}\bar{A}_{\Delta d}^t\mathbb{P}\right\}\begin{bmatrix}x(t) \\ \sigma(t)\end{bmatrix} + \int_{t-\tau}^{t}\sigma^t(s)Q_\sigma\sigma(s)\,ds \\ &+ 2\begin{bmatrix}x(t) \\ \sigma(t)\end{bmatrix}^t\mathbb{P}^t\begin{bmatrix}0 \\ B_oK_o\, e_n\end{bmatrix}\bar{\mu} + 2\begin{bmatrix}x(t) \\ \sigma(t)\end{bmatrix}^t\mathbb{P}^t\begin{bmatrix}0 \\ B_oK_o\, e_n\end{bmatrix}\beta\end{aligned} \tag{4.31}$$

Using (4.4)-(4.5), (4.12) and (4.31), we reach

$$\begin{aligned}\dot{V}_{ut} &= \dot{V}_x + \dot{V}_\sigma + \bar{\mu}\dot{\bar{\mu}} + \beta\dot{\beta} \\ &= \pi^t(t)\begin{bmatrix}\begin{matrix}\mathbb{P}^t\mathcal{A}_{\Delta Kd} + \mathcal{A}_{\Delta Kd}^t\mathbb{P} \\ +\bar{K}_o^t\bar{K}_o + E_1RE_1^t \\ +\tau\mathbb{P}^t\bar{A}_{\Delta d}Q_\sigma^{-1}\bar{A}_{\Delta d}^t\mathbb{P} \\ +\tau E_2Q_\sigma E_2^t\end{matrix} & \cdot & \mathbb{P}^t\begin{bmatrix}0 \\ B_oK_o\, e_n\end{bmatrix} & \mathbb{P}^t\begin{bmatrix}0 \\ B_oK_o\, e_n\end{bmatrix} \\ \cdot & \cdot & \cdot & \cdot \\ \bullet & \bullet & -g & 0 \\ \bullet & \bullet & \bullet & -h\end{bmatrix}\pi(t) \\ &\stackrel{\Delta}{=} \pi^t(t)\,\Pi_n\,\pi(t),\quad \pi(t) = [x^t(t)\ \ \sigma^t(t)\ \ \bar{\mu}\ \ \beta]^t\end{aligned} \tag{4.32}$$

The following theorem summarizes the main result:

Theorem 4.3 *System (4.2) is delay-dependent stabilizable under the adaptive controller (3.8) if given a matrix* $0 < R^t = R \in \mathbb{R}^{n\times n}$*, there exist matrices* $0 < Y_x^t = Y_x \in \mathbb{R}^{n\times n}$, $Y_\sigma \in \mathbb{R}^{n\times n}$, $Y_d \in \mathbb{R}^{n\times n}$, $0 < Q_\sigma^t = Q_\sigma \in \mathbb{R}^{n\times n}$, $Z \in \mathbb{R}^{n\times n}$, $\omega \in \mathbb{R}^{n\times 1}$ *and a scalar* $\varepsilon > 0$ *satisfying the following LMI*

$$\begin{bmatrix} Y_d + Y_d^t & Y_x A_{od}^t + Z^t B_o^t - Y_d^t & 0 & 0 & 0 & Y_x N_{ad}^t & \tau Y_d^t Q_\sigma & Z & Y_x R \\ \bullet & -Y_\sigma - Y_\sigma^t + \varepsilon M M^t & \tau A_d & \omega & \omega & 0 & \tau Y_\sigma^t Q_\sigma & 0 & 0 \\ \bullet & \bullet & -\tau Q_\sigma & 0 & 0 & \tau N_d^t & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & -g & 0 & 0 & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & -h & 0 & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & -\varepsilon I & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -\tau Q_\sigma & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -I & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -R \end{bmatrix} < 0 \quad (4.33)$$

Moreover, the gain matrix is $K_0 = ZY_x^{-1}$.

Proof: Follows from a parallel development to **Theorem 4.1** and using $\omega = B_o K_o e_n$.

5. EXAMPLES

We demonstrate the theoretical developments by means of two examples.

5.1 Example 1

This example is motivated by the dynamics of bio-strata in water-quality studies on the river Nile [8]. A pilot-scale model of the form (2.1) is described by:

$$A_o = \begin{bmatrix} -0.2 & 0 & 0 \\ 0 & -0.9 & -0.3 \\ 0.8 & 0 & -1 \end{bmatrix}, \; A_d = \begin{bmatrix} 0 & 0 & 0 \\ -0.7 & 0 & 0 \\ 0 & -0.8 & 0 \end{bmatrix}, \; B_o = \begin{bmatrix} 0.8 & 0 \\ 0.2 & 0.3 \\ 0 & 0.4 \end{bmatrix},$$

$$M = \begin{bmatrix} -0.1 \\ 0.1 \\ 0.3 \end{bmatrix}, \; N_a^t = \begin{bmatrix} -0.1 \\ 0 \\ 0.1 \end{bmatrix}, \; N_d^t = \begin{bmatrix} -0.2 \\ 0 \\ 0.2 \end{bmatrix}, \; \beta = 1.5$$

The feasible solution of LMIs (4.27) attained using **Theorem 4.2** along with other related methods are gathered in Table 1

Table 1. Computational Results of Example 1

$Method$	g	ε	τ	K_o		
Theorem 4.2	0.725	1.0365	≤ 1.325	-2.146	0.157	-3.804
				0.01	0.301	-1.626
Reference [7]	0.933	1.371	≤ 0.765	-4.106	0.113	-3.421
				0.211	0.020	-1.779
Reference [8]	1.146	1.422	≤ 0.927	-3.422	0.157	-3.554
				0.176	0	-1.824
Reference [10]	0.811	1.145	≤ 1.024	-2.335	0.177	-3.662
				0.219	0.001	-1.775

A simple comparison reveals that the technique developed in this work yield larger admissible time delay factor and smaller adaptive gain.

5.2 Example 2

The following example is motivated by the dynamics of machining chatter [9] with the matrices of system (2.1) given by:

$$A_o = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -16 & 10 & 0 & 0 \\ -5 & -15 & 0.02 & -0.25 \end{bmatrix}, \; A_d = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \; B_o = \begin{bmatrix} 0.2 \\ 0 \\ 0.5 \\ 0.8 \end{bmatrix},$$

$$M = \begin{bmatrix} -0.1 \\ 0.2 \\ 0.4 \\ -0.5 \end{bmatrix}, \; N_a^t = \begin{bmatrix} 0.1 \\ 0 \\ -0.1 \\ 0 \end{bmatrix}, \; N_d^t = \begin{bmatrix} 0 \\ 0.2 \\ 0 \\ -0.2 \end{bmatrix}$$

Once again, from the results presented in Table 2, the superiority of the developed adaptive schemes of this work are noted

Table 2. Computational Results of Example 2

$Method$	g	h	ε	τ	K_o			
Theorem 4.3	0.6625	0.4255	1.4345	≤ 1.2935	-0.387	1.245	-0.336	-0.804
Reference [7]	0.7714	0.6629	1.9982	≤ 0.8652	-1.042	1.986	-0.452	-1.245
Reference [8]	0.9017	0.7511	2.0176	≤ 0.9064	-0.387	1.245	-0.765	-1.334
Reference [10]	0.7017	0.4445	1.6582	≤ 1.1147	-0.701	2.025	-0.298	-0.911

6. CONCLUSIONS

Delay-independent and delay-dependent robust stabilization schemes have been established for a class of continuous-time systems with state-delays and norm-bounded uncertainties against controller gain variations. State-feedback adaptive controllers have been constructed for the cases of known gain perturbation bounds and unknown norm-bounded perturbations. Simulation results have been presented to demonstrate the developed theory.

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Biographies

Magdi Sadek Mahmoud received B.S. (Honors) in communications engineering, M.S. in electronic engineering and Ph.D. in systems engineering from Cairo University, Egypt, in 1968, 1972, 1974, respectively. He has been a Professor of Engineering since 1984. Throughout his professional career, he worked as academic staff at Al-Azhar University (1968-1971); Cairo University (1971-1992); the American University in Cairo (1973-1992); the Egyptian Air Academy (1986-1988); Modern Sciences & Arts University (2000-2001) and Arab Academy for Sciences and Technology (2001-2002) (Egypt); Kuwait University (1981-2000) and Kuwait Institute for Scientific Research (KISR) (Kuwait) (1988-1989); UMIST (1980-1981) (UK), Pittsburgh University and Case Western Reserve University (1975-1977) (USA), Nanyang Technological University (1997-1998) (Singapore), The University of South Australia (2001) (AUSTRALIA) and UAE University, Al-Ain (2002-2004) (UAE). For more than thirty years, Dr. Mahmoud has taught various academic courses within the electrical, electronic, computer, systems and industrial engineering curricula as well as applied mathematics at both undergraduate and postgraduate levels of institutions worldwide. He has also offered many intensive and specialized training courses to engineers and practitioners.He is the principal author of ten (10) books, nine (9) book-chapters and the author/co-author of more than 300-refereed papers and 45-scientific reports. He is the recipient of the 1978, 1986 Science State Incentive Prizes for outstanding research in engineering (Egypt); the 1986 Abdul-Hameed Showman Prize for Young Arab Scientists in engineering sciences, (Jordan) and the 1992 Prestigious Award for Best Researcher at Kuwait University, (Kuwait). He also holds the State Medal of Science and Arts-first class, 1979 (Egypt) and the State Distinguished Award-first class, 1995 (Egypt). He is listed in the 1979 edition of Who's Who in Technology Today (USA). As of July 5th, 2004 he took the position of Dean of Canadian International College (CIC) in Cairo.

El-Kebir Boukas was born in Morocco. He received the engineer degree in Electrical engineering in 1979 from Ecole Mohammadia d'Ingenieurs, Rabat, Morocco, and the M. Sc. and Ph. D. degrees in Electrical engineering both from Ecole Polytechnique de Montreal, Canada respectively at 1984 and 1987. He worked as an Engineer in R.A.I.D, Tangier, Morocco, from 1979 to 1980, and as a Lecturer at the University Caddy Ayyad, Marrakech, Morocco from 1980 to

1982. In 1987 he joined the Mechanical Engineering Department at Ecole Polytechnique de Montreal where he is now a full professor. His research interests include stochastic control, robust control, optimal control, modeling and control of flexible manufacturing systems, mechatronics. He is the author of three books in control theory and more than 30 invited chapters in edited books. He also edited a book on manufacturing and telecommunications. He is the author of more than 200 technical publications most of them are in control theory and manufacturing systems.

Abdulla Ismail obtained his B.Sc. ('80), M.Sc. ('83), and Ph.D. ('86) degrees, all in electrical engineering, from the University of Arizona, U.S.A. Currently, he is a Corporate Manager of Education at Dubai Silicon Oasis Authority, UAE. He spent 19 years as Professor of Electrical Engineering at UAE University, AL-Ain, UAE. He is a senior member of the IEEE and past Counselor of the IEEE Student Branch at UAE University. He is also a member of the IEEE Control Society and the IEEE Education Society. He is a recipient of the IEEE Millennium Medal, the Fulbright Scholarship, and the College of Engineering Distinguished University & Community Service Awards for 1998 and 1999. His main interests are in applied control systems, intelligent systems, desalination plants control, and power systems control. He taught courses in all areas of control engineering, intelligent systems, and applications of information technology in engineering and business. He held several administrative positions at UAE University, among which was Vice-Dean of the College of Engineering and Advisor to the Deputy Vice Chancellor for Computer Affairs. Also, he held the chairmanship and membership of several planning as well as executive committees and boards in the university and the Ministry of Higher Education and Scientific Research.

OBSERVER BASED MEASUREMENT OF THE INPUT CURRENT OF A NEURON

K. Röbenack [1,*], P. Goel [2]

[1] Institut für Regelungs- und Steuerungstheorie, Technische Universität Dresden, Germany

[2] Laboratory of Biological Modeling, National Institutes of Health (NIDDK), Bethesda, MD, USA

ABSTRACT

We propose a new technique that uses an observer to estimate the current input into a neuron whose voltage is measured electrophysiologically. As a by-product, one also obtains information about the gating variables of the ionic channels. We prove the global convergence of the observer for all voltage-gated ion channel models within the Hodgkin-Huxley formalism. The current observer can be implemented either offline or concurrently with the recording. We illustrate the workings of the observer on a well-known nonlinear neural model.

Keywords

Nonlinear Observer, Unknown Input Observer, Neuron, Electrophysiological.

1. INTRODUCTION

Neurons communicate through synapses: pre-synaptic neuronal membrane activity is transmitted across the synapse via neurotransmitters that activate post-synaptic currents to drive the neuron. Moreover, neurons exhibit subtle and complex membrane activity. This nonlinear response is influenced by both the intrinsic properties of the neuron as well as the state of the network [1, 2].

The membrane voltage of an isolated neuron can be recorded electrophysiologically, via an intracellular or extracellular electrode. This voltage change is mediated by internal ionic currents and the input current applied via the electrode. Thus, in order to understand the characteristics of neuronal activity, a neuron is stimulated with various current inputs, typically of the stepping or ramping type. Variously, the voltage of a neuron can also be clamped, i.e. held constant; in that case, currents can be measured. The ionic properties of the channels that comprise the membrane differ with the cell type. In addition, neurons of a certain type exhibit individual differences. Well-defined methods exist to characterize ion channel kinetics based on voltage- and current-clamping protocols. It is thus possible to build dynamical models of neuronal activity [3].

If a neuron in a network (tissue) is voltage-clamped, a current response is a measure of the input received by it from its pre-synaptic neighbours. Thus the effect of different inputs that approximate physiological stimuli can be studied. Unfortunately, this procedure is intrisically invasive. In particular, this technique of measurement will interfere with the neuron's activity in the network. What is required is a sensor that passively reports the current input without interfering with the voltage activity. In this paper we develop a method to determine the input current from the only available measurement, i.e., from the membrane voltage.

Most traditional experimental techniques involving electrophysiological measurements are open-loop: the neuron is stimulated with either current or voltage and the corresponding output is read-out. More recently, a feedback-based technique has become popular: the so called dynamic-clamp [4]. This allows, for example, the input current to a neuron to be modified based on the voltage output. This powerful technique has enabled the study of neuronal networks that are coupled to computer models in real-time. Dynamic clamping thus paves the way for the investigation of complex combinations of coupled organic and artificial networks. A passive observer of current is an ideal tool that can be used in combination with the dynamic clamp to measure how global stimuli applied to the system are transduced to presynaptic current patterns at various individual neurons.

Mathematical models describing membrane dynamics in neurons typically follow the formalism first described by Hodgkin and Huxley [5]. The membrane voltage is given by a system of ordinary differential equations, where the voltage dynamics is coupled to several gating variables, which describe the behavior of the ion channels in the membrane. The dynamics of the membrane voltage V is governed by

$$C\dot{V} = I - \sum_j g_j \cdot \left(V - V_j\right) \tag{1}$$

with a capacitance $C > 0$. The current I is injected into the cell, either applied via an intracellular electrode, or from pre-synaptic coupling to other cells. The sum on the right hand side of (1) represents other currents which influence the voltage dynamics, e.g. the leak current and currents flowing through ionic channels. The associated electrochemical gradients are represented by constant voltages V_j called

*Corresponding author: *E-mail:* klaus@roebenack.de

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reversal potentials. The conductance g_L associated with the leak current is constant. In case of the other currents, the conductances g_j depend on so-called gating variables w_i. More precisely, each conductance g_j usually consists of the maximal value of the conductance multiplied with non-negative integral powers of some w_i. The dynamics of these gating variables is governed by differential equations of the form

$$\dot{w}_i = \alpha_i(V)(1 - w_i) + \beta_i(V) w_i \quad \text{for} \quad i = 1, \ldots, p. \tag{2}$$

The functions $\alpha_i(V)$ and $\beta_i(V)$ are positive for all V. Eq. (2) result from a Markov model of the ith ionic channel. Each channel has two states: open and closed. The functions $\alpha_i(V)$ and $\beta_i(V)$ denote the transition rates for opening and closing, respectively. The gating variable w_i denotes the probability that the ith channel is in the open state. The number p of equations (2) depends on the selected model. The whole model (1)-(2) is a system of first order nonlinear ordinary differential equations.

From a control-theoretic point of view, system (1)-(2) is a single-input single-output state-space system with the input I and the state variables V and $w = (w_1, \ldots, w_p)^T$. The output V is measured. We propose a two-stage approach to estimate the input I. First, we design an observer to obtain the state vector w. Second, we use the information provided by the observer to obtain an estimate of the input I using a filter.

We will use an observer to estimate those quantities of (1)-(2) which are not measured directly. The problem of observer design has received significant attention during the last decades [6-8]. Classical observers provide an estimate of the state based on input and output information [8]. These observers are not applicable since the input I is not measured.

Extensions of observer theory have been made to systems with unmeasured inputs. These observers are called unknown input observers. The existence conditions for unknown input observers of linear time invariant systems are well-known [9-11]. For nonlinear systems, the existence conditions of unknown input observers are not well established. Design methods exist only for special classes of nonlinear systems. The design method proposed in [12, 13] is based on a certain decomposition of the system into two subsystems. In turns out that systems of the class (1)-(2) are already decomposed into this special form. We will employ this approach to design an unknown input observer to estimate the unmeasured state vector w.

The problem of real-time observation of an input occurs also in communication by chaotic signals [14]. In theory, we would use the inverse system approach suggested in [15, 16]. In that case, however, we have to differentiate the measured output numerically. Unfortunately, numerical differentiation by divided difference schemes in not reliable. To circumvent this problem, we design an additional low-pass filter to generate a smoothed estimate of the input.

This paper is structured as follows. In Section 2 we derive our estimation algorithm. We apply our method to a particular cell model in Section 3. The conclusions are given in Section 4.

2. OBSERVER AND FILTER DESIGN

First, we discuss the possible usage of conventional observers. Next, we design an unknown input observer of the system and show its convergence. Finally, a filter is employed to estimate the input.

2.1 Conventional Observers

The class of models described by (1)-(2) has the form

$$C\dot{V} = f(V, \mathbf{w}) + I \tag{3}$$

$$\dot{\mathbf{w}} = \mathbf{g}(V, \mathbf{w}) \tag{4}$$

with the measured output V and the unknown initial value w_0. The first subsystem (3) is 1-dimensional, whereas the dimension p of the second subsystem (4) depends on the model under consideration. Note that maps f and g are nonlinear.

In the beginning, we discuss the design of the observer to estimate the unknown state variables. In the last decades, several techniques for a systematic observer design have been developed [6-8, 17]. Most of these methods are not directly applicable because they require an explicit knowledge of the input signal. Therefore, we have to modify the model (3)-(4). A possible approach is to assume that the input signal varies slowly or changes only occasional between different regimes. In other words, we assume that the input signal is “almost” constant. This information can be incorporated into the model by an augmentation of (3)-(4) with a further differential equation $\dot{I} = 0$. The resulting $(p+2)$-dimensional model

$$\begin{aligned} C\dot{V} &= f(V, \mathbf{w}) + I \\ \dot{\mathbf{w}} &= \mathbf{g}(V, \mathbf{w}) \\ \dot{I} &= 0 \end{aligned} \tag{5}$$

is autonomous. This augmentation of the original system is a common approach in observer-based parameter estimation [18, 19]. Indeed, this idea is also used to design observers for systems with unmeasurable inputs [9]. Theoretically, one could apply arbitrary observer design methods to (5). In fact, we tried this approach in the beginning.

For linear systems, the observer design problem has been solved by Luenberger [20, 21], see also references cited in [22]. The design procedures of linear systems can be applied to the linearization of a nonlinear system, provided the systems trajectory stays in a neighbourhood of a given operating point. Unfortunately, this is not the case here, because the system shows large oscillations.

The first mathematically justified approach to design observers for nonlinear systems was developed by Thau [23]. The idea is to dominate the nonlinearities by a sufficiently large linear part in the error dynamics. The choice of the observer gain vector is not based on a local linearization but on global Lyapunov techniques [24, 25]. These design methods did not work for our systems because the large observer gains made the numerical integration of the observer’s equations utterly impossible.

The development of differential geometric methods in nonlinear control gave rise to a whole class of new observer design methods [26-30]. For all these methods, to obtain the observer gain one needs to compute certain Lie derivatives symbolically. The systems in our application are highly nonlinear. We obtained very large and complicated expressions for the observer gains (thousands of lines of C code). At best, the resulting observer did not diverge, but we were not able to extract a reasonable estimate for I.

For our point of view, conventional observer design techniques are not suitable to solve our estimation problem.

2.2 Unknown Input Observers

Now, we will design an unknown input observer. Design procedures are well-established for linear time-invariant systems [9-11]. Only a limited number of design methods exist for nonlinear systems. Our approach is based on [12, 13]. We take the structure of system (3)-(4) into account. This system is already decomposed into two subsystems. The crucial point is that the second subsystem (4) depends not explicitly on the input I. More precisely, system (3)-(4) is already in the Byrnes-Isidori normal form with relative degree one [31].

The state V of the first subsystem (3) is measured. As an observer for w we suggest a copy of subsystem (4), which is driven by the measured output:

$$\dot{\hat{\mathbf{w}}} = \mathbf{g}(V, \hat{\mathbf{w}}), \quad \hat{\mathbf{w}}(0) = \hat{\mathbf{w}}_0 \in \Re^p. \tag{6}$$

The observation error $\tilde{w} = w - \hat{w}$ is governed by the error dynamics

$$\dot{\tilde{\mathbf{w}}} = \mathbf{g}(V, \mathbf{w}) - \mathbf{g}(V, \hat{\mathbf{w}}), \quad \tilde{\mathbf{w}}(0) = \mathbf{w}_0 - \hat{\mathbf{w}}_0. \tag{7}$$

The trajectory $\hat{w}$ of the observer (6) converges to the state w of (4) for $t \to \infty$ if the equilibrium $\tilde{w} = 0$ of the error dynamics (7) is asymptotically stable uniformly in V. In other words, we assume that for all V we have $\tilde{w}(t) = w(t) - \hat{w}(t) \to 0$ as $t \to \infty$. Then, subsystem (4) is said to have a steady state solution property [32]. We will show in Section 2.3 that the class of systems discussed here poses this property. Since the state of subsystem (3) is already known by measurement, the whole system (3)-(4) is detectable [32].

In contrast to conventional observers, we have no observer gain to adjust the convergence rate of the observer (6). In so far, our observer is similar to so-called asymptotic observers known from biological and chemical process control [33, 34]. Moreover, observer (6) is a reduced observer since we reconstruct only subsystem (4). Combining the measured voltage V and the observer trajectory $\hat{w}$ yields an estimation of the whole state of (3)-(4), even though the input I is unmeasured.

2.3 Stability Analysis

We show here that the observer (6) converges globally. In particular, we also claim that this type of observer is applicable to all cell models of the Hodgkin-Huxley type [5]. In addition to the Connor-Stevens model which will be introduced in Section 3, this class of models includes several other well-known models such as the Morris-Lecar model [35], the FritzHugh-Nagumo model [36, 37], and the Traub model [38, 39], to name a few. If additional information is available, it is possible to extend the current observer we present here to other ion channels that are not just voltage-gated, but are also modulated by intracellular activity, e.g. to use the observer with a bursting model of pancreatic beta-cells [40], simultaneous measurements of calcium and adenosine triphosphate (ATP) would be required.

In our application, we consider models of the type (3)-(4), whose underlying structure is given by (1)-(2). The observer (6) is designed for the p equations of the type (2) with functions $\alpha_i(V)$ and $\beta_i(V)$. To prove the convergence of the observer we consider the difference between the original system and the observer. We show that this difference goes to zero using Lyapunov stability theory.

The equations (2) of system (4) have the form

$$\begin{aligned} \dot{w}_i &= \alpha_i(V)(1 - w_i) + \beta_i(V) w_i \\ &= \alpha_i(V) - (\alpha_i(V) + \beta_i(V)) w_i \end{aligned} \tag{8}$$

for $i = 1, \ldots, p$. The corresponding observer

$$\dot{\hat{w}}_i = \alpha_i(V) - (\alpha_i(V) + \beta_i(V)) \hat{w}_i \tag{9}$$

with the initial value $\hat{w}_i(t) \in \Re$ is excited by the measured voltage V. The observation error of the ith gating variable is defined by $\tilde{w}_i = w_i - \hat{w}_i$. The associated error dynamics is governed by

$$\dot{\tilde{w}}_i = \dot{w}_i - \dot{\hat{w}}_i = -(\alpha_i(V) + \beta_i(V)) \tilde{w}_i \tag{10}$$

with an initial value $\tilde{w}_i(0) = w_i(0) - \hat{w}_i(0)$. We use the continuously differentiable candidate Lyapunov function

$$\Upsilon(\tilde{\mathbf{w}}) = \frac{1}{2} \sum_{i=1}^{p} \tilde{w}_i^2$$

with the vector-valued argument $\tilde{w} = (\tilde{w}_1, \ldots, \tilde{w}_p)$. The function Y is positive definite since it is a quadratic form, i.e., $Y(0) = 0$ and $Y(\tilde{w}) > 0$ for all $\tilde{w} \neq 0$. Moreover, Y is radially unbounded, i.e., $Y(\tilde{w}) \to \infty$ for $\|\tilde{w}\| \to \infty$. The total derivative of $Y(\tilde{w})$ along the error dynamics (10) is calculated as

$$\dot{\Upsilon}(\tilde{\mathbf{w}}) = \sum_{i=1}^{p} \tilde{w}_i \dot{\tilde{w}}_i = -\sum_{i=1}^{p} (\alpha_i(V) + \beta_i(V)) \tilde{w}_i^2.$$

The quadratic terms are always non-negative. If $\tilde{w}$ is not the zero vector, at least one term w_i^2 is strictly positive. Moreover, we have $\alpha_i(V), \beta_i(V) > 0$ for all V by construction. (Recall that these functions are transition rates resulting from a Markov model.) Therefore, we have $\dot{Y}(\tilde{w}) < 0 \quad \forall \tilde{w} \neq 0$.

Hence, by Lyapunov's Theorem [41], the equilibrium $\tilde{w} = 0$ of (10) is globally asymptotically stable, i.e., $\tilde{w}(t) \to 0$ for $t \to \infty$ and any initial value $\tilde{w}(0) \in \Re^p$. This implies $\hat{w}(t) \to w(t)$ for $t \to \infty$, that is, the trajectory $\hat{w}(t)$ of the observer (9) converges to the trajectory $w(t)$ of the original system (8) for $t \to \infty$.

2.4 Input Estimation

Now, we make use of the information generated by the observer (6) to obtain an estimate of the current I. For known trajectories of V and w we could compute the input I exactly from (3) by

$$I = C\dot{V} - f(V, \mathbf{w}) \quad (11)$$

Since w is not available directly but estimated by the observer (6), we consider an estimate $\hat{I}$ of I defined by

$$\hat{I} = C\dot{V} - f(V, \hat{\mathbf{w}}) \quad (12)$$

For a continuous map f we have $\hat{I}(t) \to I(t)$ for $t \to \infty$ if $\hat{w}(t) \to w(t)$ for $t \to \infty$, i.e., the estimation (12) converges to the exact input (11) provided the observer (6) converges to subsystem (4). Using the inverse system approach [15, 16], one would estimate I by (12). However, we measure V but not its time derivative $\dot{V}$. A numerical computation of $\dot{V}$ from V by divided differences provides only a rough estimate of the derivative.

Up to now, we assumed that the voltage V generated from (3)-(4) is exactly known. In practical applications we also have to take disturbances such as noisy measurement into account. More precisely, we augment the exactly known voltage V by an additive disturbance signal ε. If we replace V in Eq. (12) by the measured voltage $(V+\varepsilon)$, the estimated current $\hat{I}$ would not only depend on the disturbance ε but also on its time derivative $\dot{\varepsilon}$. This is disadvantageous especially if the disturbance signal is indeed random noise.

To avoid an explicit computation of $\dot{V}$ and to attenuate the influence of the disturbance ε we use a filter. More precisely, for the right hand side of (12) we use a low-pass filter with a continuous time transfer function

$$T(s) = \frac{1}{1 + a_1 s + a_2 s^2 + \cdots + a_r s^r} \quad (13)$$

of order $r \geq 1$. The coefficients $a_1, \ldots, a_r$ have to be chosen such that all poles of (13) are in the open left half plane. In the time domain, a filter given by (13) is a linear operator. In the following, we denote the action of a filter with the transfer function T on the signal $\hat{I}$ by $T \circ \hat{I}$. The application of (13) to (12) yields the filtered signal

$$\begin{aligned} \bar{I}(t) &= T(s) \circ \hat{I}(t) \\ &= C \cdot T(s) \circ \dot{V}(t) - T(s) \circ f(V(t), \hat{\mathbf{w}}(t)). \end{aligned} \quad (14)$$

We assume that $V(t) = 0$ for $t < 0$. Between V and its time derivative $\dot{V}$ there holds $\boldsymbol{L}\{\dot{V}\} = s\boldsymbol{L}\{V\} - V(-0)$, where $\boldsymbol{L}$ denotes the Laplace transform. This results in $T \circ \dot{V} \equiv sT(s) \circ V$, i.e., instead of filtering the time derivative $\dot{V}$ by (13) we filter the measured trajectory V by

$$\frac{s}{1 + a_1 s + a_2 s^2 + \cdots + a_r s^r}. \quad (15)$$

The filtered estimate (14) is obtained by

$$\bar{I}(t) = sCT(s) \circ V(t) - T(s) \circ f(V(t), \hat{\mathbf{w}}(t)). \quad (16)$$

Taking the common denominator of (13) and (15) into account, Eq. (16) can equivalently be written

$$\bar{I}(t) = \frac{sCV(t) - f(V(t), \hat{\mathbf{w}}(t))}{1 + a_1 s + a_2 s^2 + \cdots + a_r s^r}. \quad (17)$$

In (17), the numerator degree does not exceed the denominator degree, i.e., the transfer function is proper. Hence, the filter (17) can be implemented without differentiators. The whole estimation scheme is shown in Fig. 1.

The purpose of the filter is to enhance the desired signal $\hat{I}$ relative to disturbances such as noise. Here, the filtering is done on the basis of a suppression of selected frequencies to damp interfering signals. Since the current I is nearly constant, a natural choice for the filter is a low-pass. The most important parameter of a low-bass filter is its cut-off frequency ω_o, at which the gain drops by some specified amount. Although there are many possibilities to design a low-pass filter, in most applications Butterworth, Bessel, Chebyshev and Cauer (or elliptic) filters are used [42]. For our experiments we employed a Bessel low-pass filter.

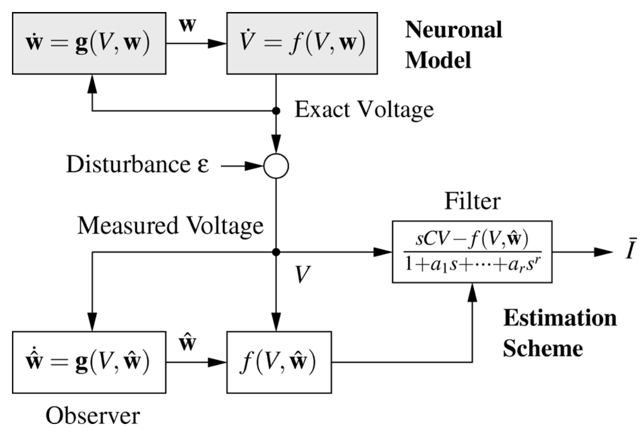

Figure 1. Reconstruction scheme for current I based on measurement of voltage *V*

3. APPLICATION TO THE CONNOR-STEVENS MODEL

We demonstrate the estimation algorithm on a cell model derived by J. A. Connor and C. F. Stevens [43]. Like the Hodgkin-Huxley model of nerve activity of the squid giant axon, the Connor-Stevens model describes important aspects of the biophysical behaviour of gastropod neuron somas. Here, in addition to the delayed-rectifier potassium, fast sodium and leak currents as in the Hodgkin-Huxley model, there is also an A-type potassium current. It is a well-studied model of Type 1 excitability: its periodic activity is the result of a saddle node bifurcation at current threshold [44, 45]. Several neurons, including the regular-spiking neurons of the somatosensory cortex display Type 1 behavior. For a more complete discussion of neuronal activity from the point of view of bifurcation theory we point to [2, 46].

The voltage dynamics read as

$$C\dot{V} = I - I_{Na} - I_K - I_L - I_A \quad (18)$$

with the input current I and $C = 1\mu F/cm^2$. The model has a leak current I_L, one current I_{Na} for the sodium ions (Na^+), and two currents I_K and I_A for the potassium ions (K^+). These currents are given by

$$\begin{aligned} I_{Na} &= g_{Na} m^3 h (V - V_{Na}) \\ I_K &= g_K n^4 (V - V_K) \\ I_L &= g_L (V - V_L) \\ I_A &= g_A a^3 b (V - V_A) \end{aligned} \tag{19}$$

with the conductances $g_K = 20 mS/cm^2$, $g_{Na} = 120 mS/cm^2$, $g_L = 0.3 mS/cm^2$, $g_A = 47.7 mS/cm^2$, the reversal potentials $V_{Na} = 55 mV$, $V_K = -72 mV$, $V_L = -17 mV$, $V_A = -75 mV$, and the dimensionless gating variables m, h, n, a, b. The equivalent circuit representation of Eqs. (18) and (19) is shown in Fig. 2.

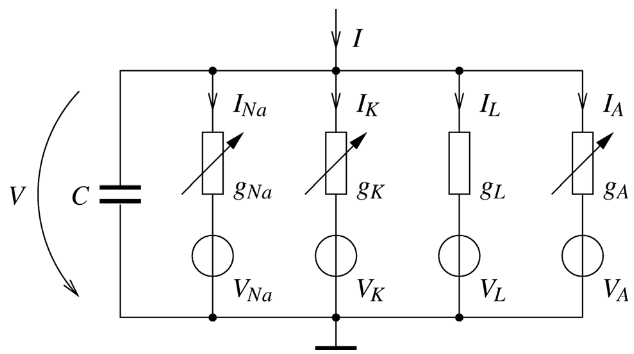

Figure 2. Equivalent circuit representation of the Connor-Stevens model (19)

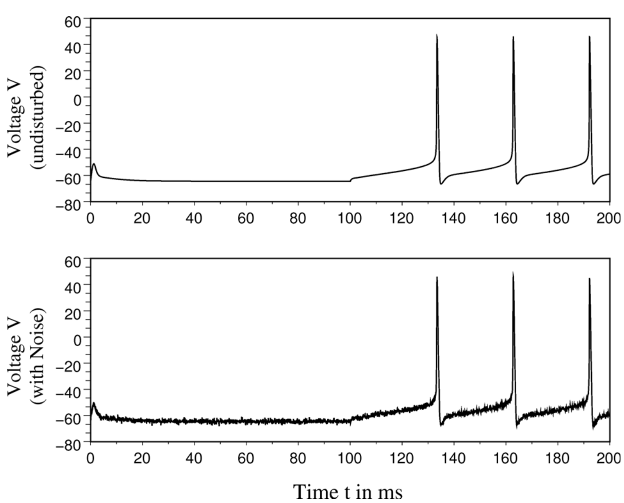

Figure 3. Output voltage generated from the Connor-Stevens model without and with measurement noise

The gating variables m, h, n influence the ionic currents I_{NA} and I_K. The additional ionic current I_A depends on the gating variables a and b. Although both currents I_K and I_A are carried by potassium ions, the model does not require that the reversal potentials V_K and V_A are equal. The gating variables are governed by the differential equations

$$\begin{aligned} \dot{m} &= \alpha_m(V)(1-m) - \beta_m(V)m \\ \dot{h} &= \alpha_h(V)(1-h) - \beta_h(V)h \\ \dot{n} &= \alpha_n(V)(1-n) - \beta_n(V)n \\ \dot{a} &= \frac{a_\infty(V) - a}{\tau_a(V)} \\ \dot{b} &= \frac{b_\infty(V) - b}{\tau_b(V)} \end{aligned} \tag{20}$$

with the functions

$$\begin{aligned} \alpha_m &= \frac{0.38(V+29.7)}{1-\exp(-0.1(V+29.7))} \\ \alpha_h &= 0.266\exp(-0.05(V+48)) \\ \alpha_n &= \frac{0.02(V+45.7)}{1-\exp(-0.1(V+45.7))} \\ \beta_m &= 15.2\exp(-0.0556(V+54.7)) \\ \beta_h &= \frac{3.8}{1+\exp(-0.1(V+18))} \\ \beta_n &= 0.25\exp(-0.0125(V+55.7)) \end{aligned}$$

and

$$\begin{aligned} a_\infty &= \left(\frac{0.0761\exp(0.0314(V+94.22))}{1+\exp(0.0346(V+1.17))}\right)^{1/3} \\ \tau_a &= 0.3632 + \frac{1.158}{1+\exp(0.0497(V+55.96))} \\ b_\infty &= \left(\frac{1}{1+\exp(0.0688(V+53.3))}\right)^4 \\ \tau_b &= 1.24 + \frac{2.678}{1+\exp(0.0624(V+50))}. \end{aligned}$$

The first three equations of (20) are already in the form (2), and the last two equations of (20) can easily be rewritten into (2).

For the simulation we use the initial values $V(0) = -64.453 mV$, $m(0) = 0.0159$, $h(0) = 0.9437$, $n(0) = 0.196$, $a(0) = 0.0559$, $b(0) = 0.2175$ and the current signal

$$I(t) = \begin{cases} 5\,mA & \text{for} \quad 0\,ms \le t \le 100\,ms \\ 10\,mA & \text{for} \quad t > 100\,ms. \end{cases} \tag{21}$$

This signal can be interpreted as follows: For $t = 0, \ldots, 100\ ms$, a low value of background activity leaves the neuron close to its resting state. At $t = 100\ ms$, a stimulus arrives at the neuron and kicks the neuron with an excitation and induces a repetitive firing. From a mathematical point of view, this qualitative change in the system's behaviour is due to a saddle-node bifurcation that gives rise to oscillations emerging with arbitrarily low frequencies. The resulting oscillations are shown on the top of Fig. 3.

The measured voltage of the Connor-Stevens model (18)-(20) is used to reconstruct the other state variables *m*, *h*, *n*, *a* and *b*. The unknown input observer (6) consists of a copy of Eqns. (20), which are driven by the measured voltage *V* of (18):

$$\begin{aligned}
\dot{\hat{m}} &= \alpha_m(V)(1-\hat{m}) - \beta_m(V)\hat{m} \\
\dot{\hat{h}} &= \alpha_h(V)(1-\hat{h}) - \beta_h(V)\hat{h} \\
\dot{\hat{n}} &= \alpha_n(V)(1-\hat{n}) - \beta_n(V)\hat{n} \\
\dot{\hat{a}} &= \frac{a_\infty(V) - \hat{a}}{\tau_a(V)} \\
\dot{\hat{b}} &= \frac{b_\infty(V) - \hat{b}}{\tau_b(V)}
\end{aligned} \tag{22}$$

Since we have no further knowledge of these variables at $t = 0$, we use the zero vector of $\Re^5$ as an initial value of the observer (22). The simulation was performed with Simulink®, where the two subsystems of (18)-(20) are implemented as so-called S-functions [47]. The simulation results shown in Fig. 4 indicate that the unknown input observer (22) converges.

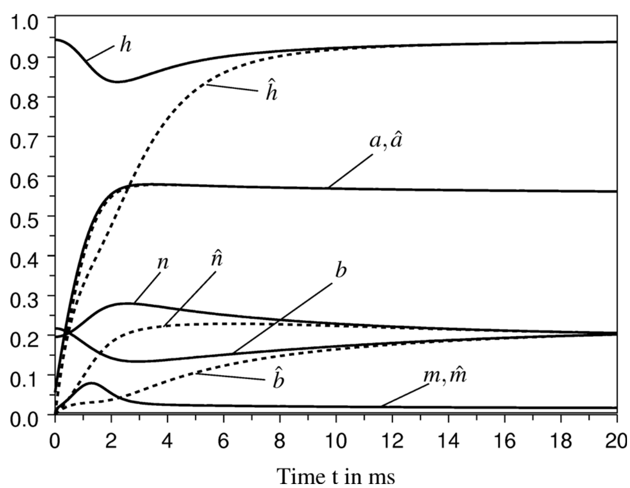

Figure 4. Trajectories of Connor-Stevens model and the unknown input observer (22)

In addition to the ideal case of an undisturbed voltage measurement we also consider the perturbed case. In particular, to the output voltage *V* of (18) we add band-limited discrete time white noise with sample time $0.1 ms$ and power 0.1. The output voltages with and without noise are shown in the aforementioned Fig. 3. To estimate the current *I* by (16) we need to choose a low-pass filter and the poles of the transfer function (13). For the suppression of the artificially introduced measurement noise we use a 4th order Bessel filter. First, the filter is designed with a cut-off frequency $\omega_o = 1$ *rad/ms*. The filtered current $\tilde{I}$ for the unperturbed case and for five realizations of random output perturbations is shown on the top of Fig. 5. Although the increase of *I* from $5mA$ to $10mA$ at $t = 100$ *ms* can be deduced from a visual inspection, the level of the perturbations is not yet satisfying. For a better suppression of the noise we decrease the cut-off frequency of the filter to $\omega_o = 0.1$ *rad/ms*. The result is also shown on the bottom in Fig. 5. As expected, we obtain relatively smooth curves for $\tilde{I}$. The drawback of a lower cut-off frequency is a slower transient behaviour. In general, after some transients we obtain a good estimate $\tilde{I}$ of *I*. However, we still have some deviations from the exact values of *I* at $t \approx 145$, $t \approx 170$, and $t \approx 200$ *ms*. At this point we should recall that the combination of the nonlinear unknown input observer (6) with the linear filter (13) yields a nonlinear filtering scheme.

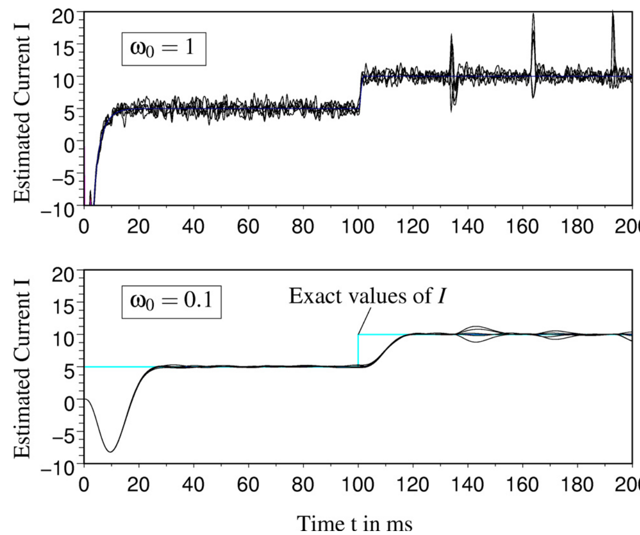

Figure 5. Estimated current $\tilde{I}$ for the Connor-Stevens model

4. CONCLUSIONS

A direct measurement of the synaptic input driving a neuron, especially in vivo, is a challenging problem. We have presented here an observer based technique useful for determining the current input into a neuron whose membrane voltage is measured directly. Observers for synaptic current could be used to implement a sensor with a minimal of interference in vivo. Theoretically, they solve the inverse problem of determining the input from the measured output. Additionally, the observer also recovers the time courses of the gating variables which cannot be directly measured. Such a capability clearly enlarges the scope of useful information that can be obtained from electrophysiological recordings. An observer for synaptic current can be used quite generally in any context that a neuron is recorded from, and especially effectively in studying small networks. They can thus have a variety of practical applications.

During the last decades, several conductance based neural models have been developed after the fashion of Hodgkin and Huxley [5]. On the other hand, mathematical analyses of the dynamical properties of neurons continue to provide considerable understanding of the behavior of neuronal networks from a theoretical point of view. Current observers can potentially be used for a direct and independent verification of theoretical models. The technique of effective current observers promises to bridge the gap between speculative theoretical modeling and direct experiment even further.

The observer design procedure employed in this paper is based on two assumptions. On the one hand, the system must be transformable into (3)-(4). By this, the system is decomposed into two subsystems, where the state of the first subsystem is known from measurement and the second subsystem does not explicitly depend on the input signal. On the other hand, the second subsystem must have a special stability property. If these assumptions hold, our design method can also be applied to other systems outside of cell biology.

The stability result of the observer is based on the assumption that the voltage is observed noiselessly. We showed by simulation that the suggested estimation scheme also works under noisy measurement, even though the estimated current signal does not have the same quality as in the unperturbed case. The attenuation of these disturbances as well as the adaption of model parameters will be subject of further research.

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Biographies

Klaus Röbenack was born in Halle/Saale, Germany in 1967. He received his Dipl.-Ing. and Dr.-Ing. degrees in electrical engineering from the Technische Universität Dresden in 1993 and 1999, respectively. Additionally, he received the Dipl.-Math. degree with honours in 2002 and the university teaching qualification (Dr.-Ing. habil.) in 2005. Dr. Röbenack is currently associated with the Institut für Regelungs- und Steuerungstheorie at Technische Universität Dresden. His research interests include nonlinear control, observer design and scientific computing.

Pranay Goel received his B. Tech. in Engineering Physics from the Indian Institute of Technology, Mumbai, India in 1998. He also received his M.S. and Ph.D.degrees in Physics from The University of Pittsburgh in 2003. Following that, he worked as a Postdoctoral Researcher at the Mathematical Biosciences Institute, The Ohio State University. He is currently a Research Fellow at the National Institutes of Health. He has worked on a number of problems in mathematical biology. His interests include dynamical systems modeling in neuroscience, cell physiology, and recently, diabetes.

SIMPLIFIED SPACE VECTOR PWM ALGORITHM FOR THREE-LEVEL INVERTER WITH NEUTRAL POINT POTENTIAL CONTROL

D. Lalili [1, 2] *, E. M. Berkouk [2], F. Boudjema [2], N. Lourci [1, 2]

[1] Laboratoire de Modelisation en Electrotechnique, Engineering Science Faculty, University of Jijel, Algeria

[2] Laboratoire de Commande des Processus, Polytechnic National School, Algeria

ABSTRACT

In this paper, we present an algorithm for the space vector pulse width modulation (SVPWM) applied to three-level diode clamped inverter. In this algorithm, the space vector diagram of the three-level inverter is decomposed into six space vector diagrams of two-level inverters. This idea allows us to generalize the two-level SVPWM algorithm into the case of three-level inverter. The redundant vectors of the space vector diagram of three-level inverter are used to control its neutral point potential using a closed loop.

Keywords

Three-level Inverter, Space Vector Pulse Width Modulation (SVPWM), Neutral Point Potential Control.

1. INTRODUCTION

The standard two-level voltage source inverter is composed of only one switching cell per phase. So, in the field of high power drive systems, the level of dc-bus voltage constitutes an important limitation on the handled power. On the other hand, the very high *dv/dt* generated with high dc-link voltage is responsible for the electromagnetic interference and motor winding isolation stress [1].

Multi-level converters offer an approach to solve these problems. In this kind of converters, the output voltage can take several discrete levels of equal magnitude. The multilevel inverter, first proposed in [2], was aimed to reduce the harmonic content of generated voltage and current waveforms. If compared with a two-level waveform, the harmonic content of such a waveform is greatly reduced. The performance of multilevel inverter depends mainly on the pulse width modulation method that used. Various multilevel pulse width modulation strategies have been developed and studied [3-6]. Among these strategies, space vector pulse width modulation (SVPWM) is more suitable for digital implementation and switching waveform optimization.

Several works apply the space vector modulation to multilevel inverters [7-10]. These works use a typical method that approximates the output voltage using the three nearest output vectors. When the reference vector changes from one region to another, it may cause intense change in output voltage. In addition, we need to calculate the switching sequences and switching time of the states at every switching period. Thus the computation time increase with the increasing number of the reference vectors. This is a main limitation of the application of this typical SVPWM.

Recently, several attempts have made to explore a simple, fast and generally applicable multilevel space vector modulation algorithm. One of the new trends is to convert the SVPWM equations into a new form using an appropriate co-ordinate system [11-15]. In [11], a method of SVPWM for high level inverters that represents output vector in three-dimensional Euclidean space is presented. The method is based on the fact that increasing the number of levels by one always forms an additional hexagonal ring of equilateral triangles, which surrounds the outermost hexagon. In [12], the used method transforms the space vector diagram from Cartesian coordinates system to 60° coordinate system. In [13], sum manipulations allow to simplify space vector diagram of three-level inverter into space vector diagram of two-level inverter. In [14], the hexagon representing space vector diagram is flatten and the reference voltage vector is normalized in order to reduce computations of the algorithm. The method used in [15] adds to SVPWM a predictive current control loop. The load current is predicted for all output voltage vectors of the inverter. The current error is calculated and the switching state that ensures the smallest value of this error is selected.

Although these methods propose general SVPWM algorithms for multilevel inverter, the coordinate transformations used in these algorithms are somewhat complicated. In this paper, we present a simple and fast modulation algorithm based on geometrical considerations in the case of three-level inverter. Using the idea given in [13], and adding simplifications and best geometrical presentation, the space vector diagram of a three-level inverter is decomposed to six space vector diagram of a two-level inverter. The computational effort using the proposed method and the complexity of the algorithm are reduced compared with other conventional space vector modulation techniques. This algorithm is general and can be used in any high-level inverter, as made in [16] in the case of five-level inverter.

One important problem associated with three-level inverter is the neutral point potential variation. The current in neutral point produce ripple in the capacitor voltages. This ripple may cause low-order harmonic contents in the output voltages. The unbalance of dc-voltages will also cause excessive voltage pressure on switches.

Several methods are proposed to suppress the unbalance of neutral point potential, generally using redundant vectors.

*Corresponding author: *E-mail:* lalilidjaafer@yahoo.fr

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Some of these methods are based on adding a zero sequence or a dc-offset to output voltage [17, 18]. In [19, 20], power electronics circuitry is added to redistribute charges between capacitors. A method based on minimizing a quadratic parameter that depends on capacitor voltages is presented in [21]. This quadratic parameter is positively defined and reach zero when the two capacitors have the same voltage. Some other works use a converter-inverter cascade [22], and apply automatic control methods, such as fuzzy logic control [23] or sliding mode control [24] to this cascade. The drawback of these methods is either high costs and system complexity, or the use of open loop scheme. In this work we use a simple and closed loop method which makes a continuous measurement of output current and difference between capacitor voltages, and chooses the redundant vector on the basis of these measurements.

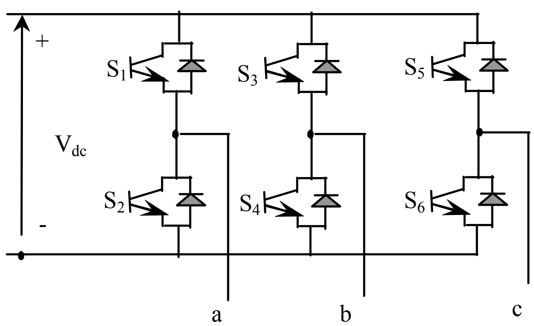


Figure 1. Two-level inverter structure

In this paper, firstly we present space vector modulation for two-level inverter and space vector diagram for three-level inverter. Subsequently, we present a simplified space vector modulation by noting that space vector diagram of three-level inverter is equivalent to six space vector diagrams of two-level inverter. Next, we present a neutral-point potential balancing algorithm based on real time measurement of load current and input dc-voltages. The proposed methods are verified through simulation.

2. SPACE VECTOR MODULATION FOR TWO-LEVEL INVERTER

Fig. 1 shows structure of two-level inverter. Each one of the three phases of the inverter has two switches and two freewheeling diodes. The inverter is supplied from a dc-voltage V_{dc}. The output phase voltages are v_a, v_b and v_c. For each phase, we define a switching signal as:

$$F_i = \begin{cases} 1 \text{ if the higher switch is on} \\ 0 \text{ if the lower switch is on} \end{cases} \quad \text{i = a, b or c} \tag{1}$$

Depending on the values of switching signals F_a, F_b and F_c, the two-level inverter has eight states, summarized in table 1, where it is also indicated the output voltage vector produced in each state. These output vectors are shown on the space vector diagram of Fig. 2, together with an arbitrary reference vector V^*, to be generated by the inverter. Note that in addition to the six non zero vectors produced in states 1 through 6 of the inverter, two zero vectors, v_0 and v_7 are also indicated on the diagram. These correspond to states 0 and 7, see Table 1.

Only vectors v_0 through v_7 can be produced at a given instant of time. Therefore, vector V^* represents an average rather than an instantaneous value, the average being taken over a period of the so called switching interval, T_s, which constitutes a small fraction of switching period, T, of the inverter. Therefore we can write:

$$T_s = \frac{T}{n} \tag{1}$$

where n is the number of switching interval per period. The switching interval in the center of which the reference voltage is located, is shown in Fig. 2 as the shaded segment.

The non-zero vectors v_1 through v_6, divide the period cycle into six, 60^o-wide sectors. The desired voltage vector, V^*, located in a given sector, can be generated by a linear combination of the two adjacent base vectors, v_x and v_y, which are framing the sector, and either one of the two zero vectors:

Table 1. States of two-level inverter

State	F_a	F_b	F_c	Voltage vector
0	0	0	0	v_0
1	0	0	1	v_1
2	0	1	0	v_2
3	0	1	1	v_3
4	1	0	0	v_4
5	1	0	1	v_5
6	1	1	0	v_6
7	1	1	1	v_7

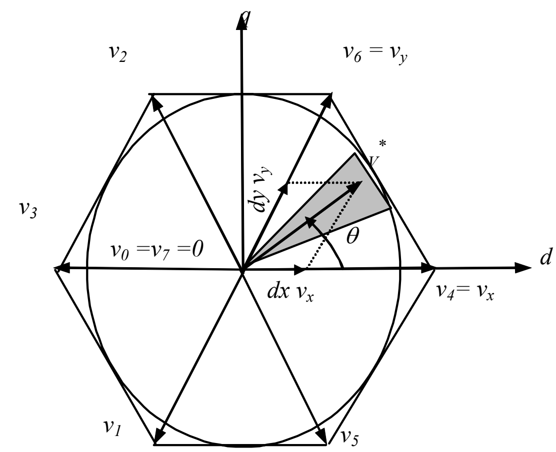


Figure 2. Space vector diagram of two-level inverter

$$V^* = d_x v_x + d_y v_y + d_z v_z \tag{3}$$

where v_z is the zero vector, while d_x, d_y, and d_z denotes the so called duty ratios of states X, Y, and Z of the inverter within the switching interval, respectively. For example, the reference voltage vector V^* in Fig.2 is located within a sector in which $v_x = v_4$ and $v_y = v_6$, hence it can be produced by an appropriately timed sequence of states $X = 4$, $Y = 6$, and $Z = 0$ or 7 of the inverter.

The duty ratios d_x, d_y and d_z of states X, Y and Z are calculated as [25]:

$$d_x = m.\ \sin(\pi/6-\theta)$$
$$d_y = m.\ \sin(\theta) \quad (4)$$
$$d_z = 1 - d_x - d_y$$

where m is the modulation index, adjustable within the 0 to 1 rang, and θ denotes the angular position of vector V^* inside the sector.

Due to the freedom of choice of the zero-vector states Z, various state sequences can be enforced in a given sector. Efficient operation of the inverter is obtained when the state sequences in consecutive switching intervals are

$$\left|X - Y - Z\right|Z - Y - X\left|\ldots\right. \quad (5)$$

where $Z = 0$ in sectors v_6-v_2, v_3-v_1 and v_5-v_4, and $Z = 7$ in the remaining sectors

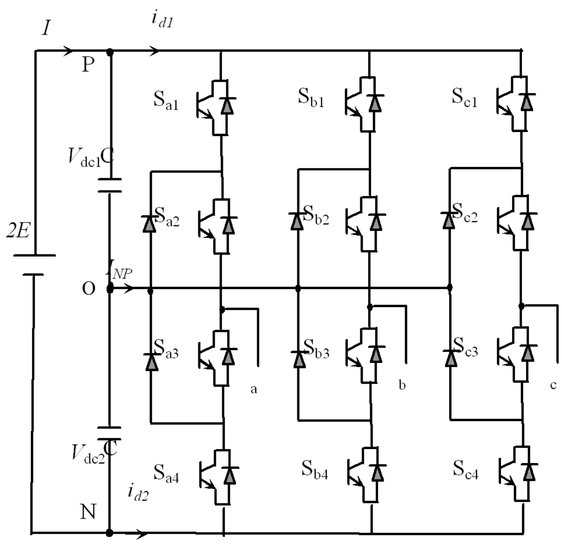


Figure 3. Three-level inverter structure

3. THREE-LEVEL INVERTER CIRCUIT

A three-level diode clamped inverter is shown in Fig. 3. Each leg is composed of two upper and lower switches with anti-parallel diodes. Two series dc-link capacitors split the dc-bus voltage in half, and six clamping diodes confine the voltages across the switches within the voltages of the capacitors. The necessary conditions for the switching states for the three-level inverter are that the dc-link capacitors should not be shorted, and the output current should be continuous. As indicated in Table 2, each leg of the inverter can have three possible switching states, P, O, or N. When the top two switches S_{x1} and S_{x2} ($x = a, b, c$) are turned on, switching state is P. When the medium switches S_{x2} and S_{x3} are turned on switching state is O. When the lower switches S_{x3} and S_{x4} are turned on, the switching state is *N*.

Fig. 4 shows the space vector diagram for three-level inverter. The output space vector is identified by combination of switching states P, O or N of the three legs. For example, in the case of PON, the output terminals *a*, *b* and *c* have the potentials E, 0, and –E respectively. Since three kinds of switching states exist in each leg, three-level inverter has (3^3 = 27) switching states, as indicated in the diagram. The output voltage vector can take only 18 discrete positions in the diagram because some switches states are redundant and create the same space vector. In Fig. 4, it is also indicated an arbitrary reference vector V^*, to be generated by the inverter.

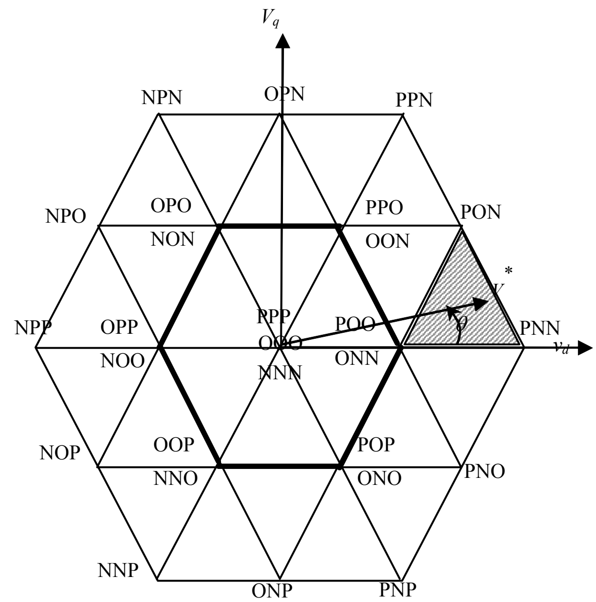


Figure 4. Space vector diagram of a three-level inverter

Table 2. States of a three-level inverter (*x* = *a*, *b*, or *c*).

Switching Symbols	Switching States				Output Voltage
	S_{x1}	S_{x2}	S_{x3}	S_{x4}	
P	ON	ON	OFF	OFF	E
O	OFF	ON	ON	OFF	0
N	OFF	OFF	ON	ON	-E

4. SIMPLIFIED SVPWM FOR THREE-LEVEL INVERTER

The space vector diagram of multilevel inverter can be divided into different forms of sub-diagrams, in such a manner that the space vector modulation becomes more simple and easy to implement, as made in several works [11-15]. But these works do not reach a generalization of the two-level SVPWM to the case of multilevel inverters; either they divide the diagram into triangles, or into interfered geometrical forms. In this work, we present a simple and fast method that divides the space vector diagram of three-level inverter into six small hexagons. Each hexagon is space vector diagram of two-level inverter, as shown in Fig. 5.

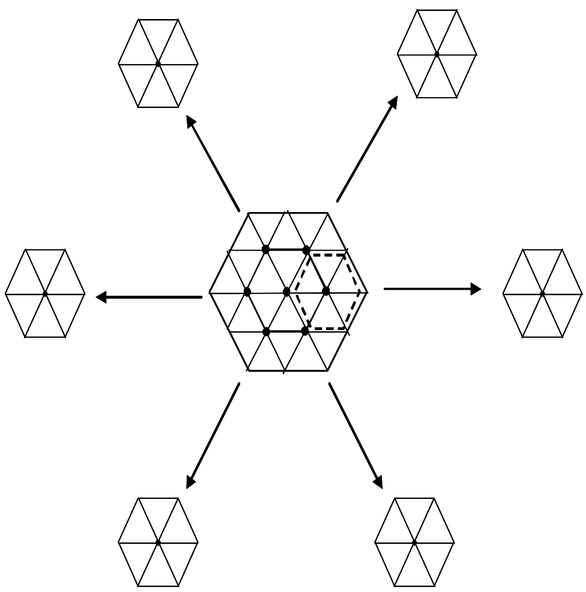


Figure 5. Decomposition of space vector diagram of a three-level inverter to six hexagons

Doing so, the space vector modulation of three-level inverter becomes very simple and similar to that of conventional two-level inverter space vector modulation. This method can be used also for higher level inverters, as we made in [16] for the case of five-level inverter. To reach this simplification, two steps have to be done. Firstly, from the location of a given reference voltage, one hexagon has to be selected among the six hexagons. Secondly, we translate the origin of the reference voltage vector towards the centre of the selected hexagon. These steps are explained in the next section.

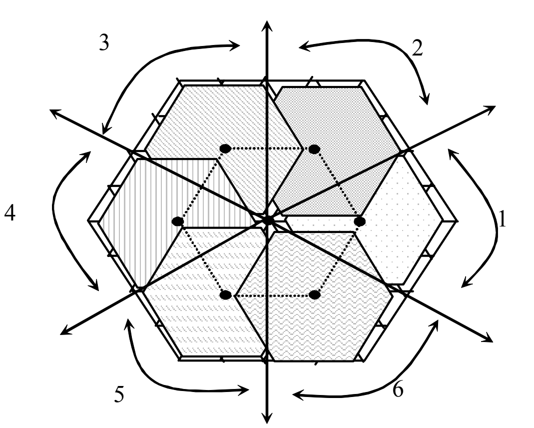


Figure 6. Division of overlapped regions

4.1 Correction of reference voltage vector

Having the location of a given reference voltage vector V^*, one hexagon is selected among the six small hexagons of the three-level space vector diagram. There exist some regions that are overlapped by two adjacent small hexagons. These regions will be divided in equality between the two hexagons as shown in Fig. 6. Each hexagon is identified by a number s defined as in equation (6).

$$s = \begin{cases} 1 & \text{if } \frac{-\pi}{6} < \theta < \frac{\pi}{6} \\ 2 & \text{if } \frac{\pi}{6} < \theta < \frac{\pi}{2} \\ 3 & \text{if } \frac{\pi}{2} < \theta < \frac{5\pi}{6} \\ 4 & \text{if } \frac{5\pi}{6} < \theta < \frac{7\pi}{6} \\ 5 & \text{if } \frac{7\pi}{6} < \theta < \frac{3\pi}{2} \\ 6 & \text{if } \frac{3\pi}{2} < \theta < \frac{11\pi}{6} \end{cases} \quad (6)$$

θ denotes the angular position of vector V^*, indicated on Fig. 4.

After selection of one hexagon, we make a translation of the reference vector V^* towards the center of this hexagon, as indicated in Fig. 7. This translation is done by subtracting the center vector of the selected hexagon from the original reference vector. Table 3 gives the components d and q of the reference voltage V^{2*} after translation, for all the six hexagons. The index (^{2}) or (^{3}) above the components indicate two or three-level cases respectively.

Table 3. Correction of reference voltage vector

S	V^{2*}_d	V^{2*}_q
1	V^{3*}_d-2.E.cos(0)	V^{3*}_q-2.E.sin(0)
2	V^{3*}_d-2.E.cos(π/3)	V^{3*}_q-2.E.sin(π/3)
3	V^{3*}_d-2.E.cos(2π/3)	V^{3*}_q-2.E.sin(2π/3)
4	V^{3*}_d-2.E.cos(π)	V^{3*}_q-2.E.sin(π)
5	V^{3*}_d-2.E.cos(4π/3)	V^{3*}_q-2.E.sin(4π/3)
6	V^{3*}_d-2.E.cos(5π/3)	V^{3*}_q-2.E.sin(5π/3)

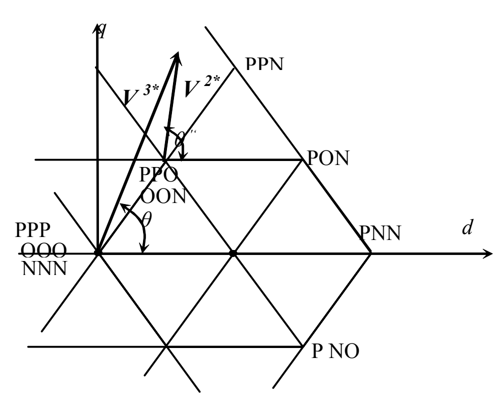


Figure 7. Translation of reference voltage vector

4.2 Sequence of switching states

Once the corrected reference voltage V^{2*} and the corresponding hexagon are determined, we can apply the conventional two-level space vector PWM to the inverter, as explained in Section 2. The reference voltage vector V^{2*} is approximated using the nearest three states, which are nodes of the triangle containing the vector, identified as X, Y, and Z. For example, in the case of Fig. 4, X is the state PNN, Y is the state PON, while Z is either the state POO or the state ONN.

The optimum sequence of these three states is selected so as to minimize the total number of switching transitions and fully optimize the harmonic profile of the output voltage. Note that from two-level space vector modulation theory, it is well known that these sequences should be reversed in the next switching interval for minimum harmonic impact [26, 27].

5. RESULTS AND DISCUSSIONS

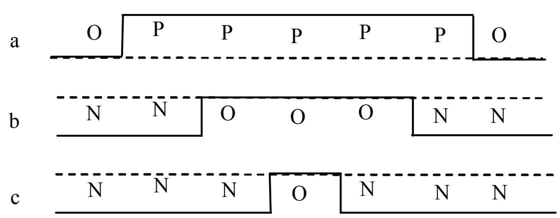


Figure 8. Outputs waveforms for sequence ZXYZYXZ

5.1 Effect of state sequences

In order to show the influence of the choice of the state sequences on the harmonics of the output voltage, we simulate the space vector algorithm for two state sequences.

Firstly, we consider the sequence ZXYZYXZ. This sequence means that in each triangle of the reference voltage vector diagram, the reference voltage vector is composed using the nodes of the triangle in turn and in reverse. For example, if the output voltage is in the hashed triangle of Fig.4, the sequence of the output voltage is: (ONN) - (PNN) - (PON) - (POO) - (PON) - (PNN) - (ONN). This sequence produces the waveforms of phase output voltages indicated on Fig. 8.

Secondly, we simulate the sequence ZOZXYZXYZZO that, other than the three node-vector of the triangle containing the reference vector, add the state NNN (noted as ZO), which is the centre of the space vector diagram, in order to synthesize the reference vector: (NNN) - (ONN) - (PNN) - (PON) - (POO) - (PNN) - (PON) - (ONN) – (NNN). Starting and ending state is NNN in every switching interval. The state NNN is added to avoid the problem of the abrupt change in output vector when passing from a triangle to another.

We simulate the association of the three-level inverter with an induction motor. Table 4 gives the simulation parameters of the inverter and the motor. The simulated output voltage and its harmonics spectrum for the proposed sequences are given in Fig. 9 and 10. We show that the harmonics of the output voltage are centred on multiples of 50.*n* frequencies, where *n* is the number of switching intervals per period of the output voltage. We show this clearly when we make zoom of the spectrum. By comparing the results given by the two sequences, we show that the sequence (ZXYZXYZ) gives best performances, because the amplitudes of the lowest harmonics are most reduced in this case. The sequence Z_OZXYZYXZZ_O gives less quality of harmonics spectrum, but this is acceptable because the main goal of this sequence is to avoid abrupt changing of switching states.

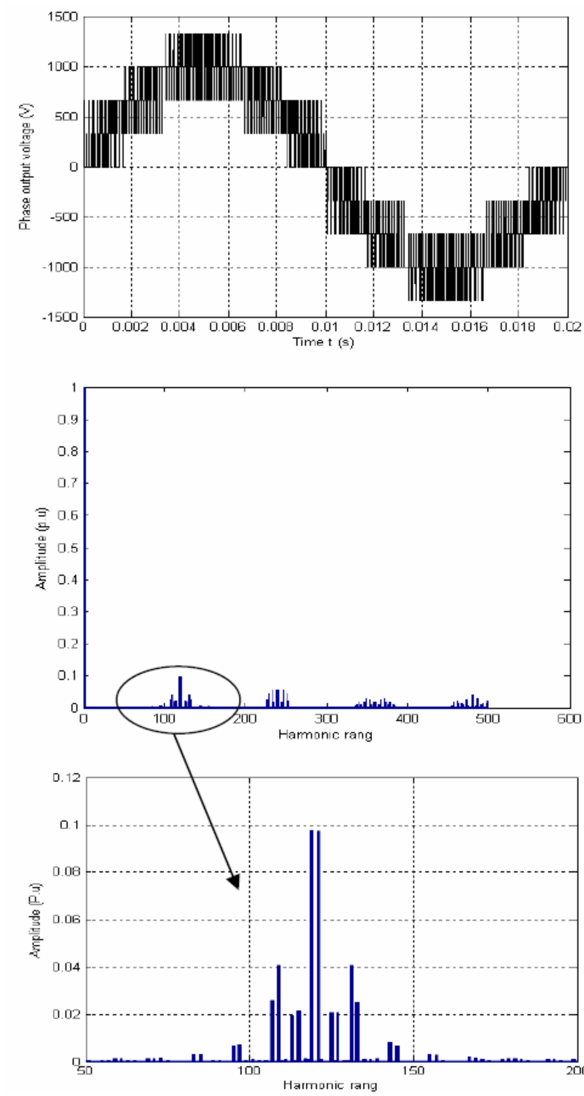


Figure 9. Output phase voltage and its harmonics spectrum for sequence ZXYZYXZ

Table 4. Simulation parameters

SVPWM parameters	Modulation index m=0.8 DC supply voltage E =800 V DC supply capacitance: C = 0.08 F Number of switching intervals *n*=120
Induction motor parameters	R_s = 3.085 Ω; R_r = 4.85 Ω ; l_s = 0.274 H; l_r = 0.274 H; l_m = 0.258 H; p = 2; *f* = 50 Hz

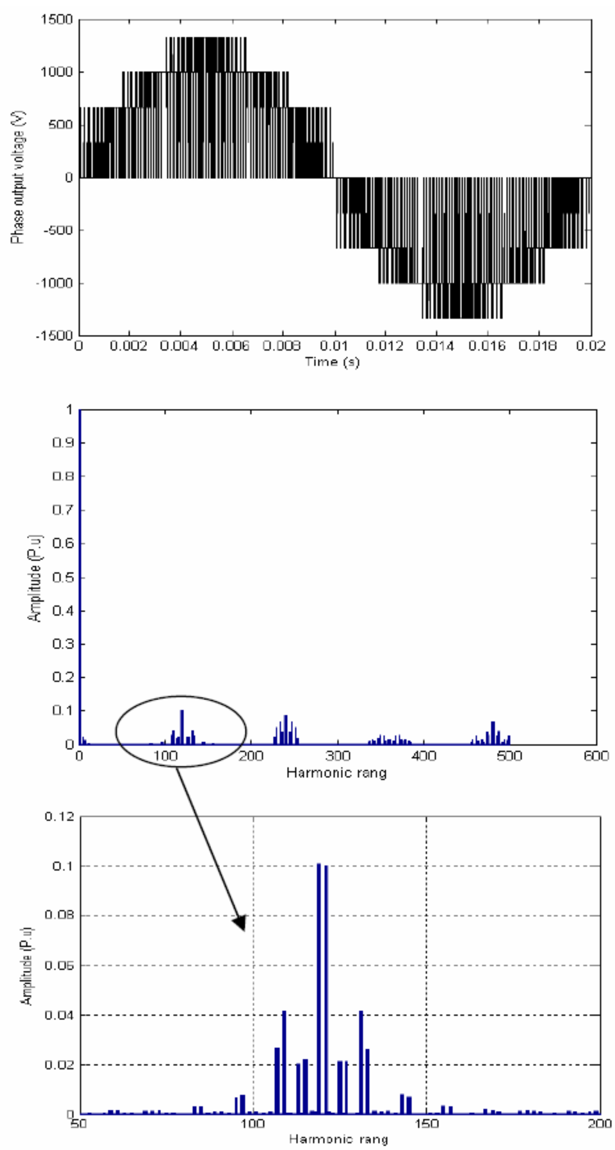


Figure 10. Output phase voltage and its harmonics spectrum for sequence Z_OZXYZYXZZ_O

5.2 Effect of number of switching intervals

The pulse width modulation is characterized by the switching frequency being higher than the output frequency. The higher the ratio of switching frequency to output frequency, the higher is the quality of the output voltage so obtained. This ratio is proportional to the number of switching intervals n.

To show the effect of number of switching intervals on the output voltage of the inverter, we simulate the space vector modulation for $n = 60$, $n = 120$, and $n = 210$. Simulation results are given in Fig.11. We sea clearly that the quality of output voltage is better for higher values of n. However, high switching frequencies results in proportionately high switching losses in the inverter switches. Therefore, the switching frequency must represent a reasonable trade-off between the quality of output waveforms and the efficiency of the inverter.

5.3 Analysis of the neutral point potential variations

The mean problem associated with three-level inverter is its neutral point potential variation. Under certain conditions, such as acceleration or deceleration, the current flowing in this neutral point causes variation of its potential.

In the space vector diagram of three-level inverter (Fig. 4), we can distinguish four types of vectors: large vectors, medium vectors, small vectors and zero vectors. The large vectors are the vectors that all of the three legs are connected to either point P or N, except in the case of all the three being connected at the same point. There are six large vectors in the space vector diagram: PNN, PPN, NPN, NPP, NNP and PNP. The medium vectors are the ones that only one phase is connected to point O and other two phases are connected to P and N each other. There are six medium voltages: PON, OPN, NPO, NOP, ONP and PNO. The small vectors are those vectors that have two phases connected at the same point. There are twelve of them: PPO, OON, OPO, NON, OPP, NOO, OOP, NNO, POP, ONO, POO and ONN. The zero vectors are the vectors that have all three phases are connected at the same point. There are three zero vectors: PPP, OOO and NNN.

To show the effect of each type of vectors on the neutral point potential, we present the load connections of one example of each type in Fig.12. It may be easily deduced from Fig.12.a and Fig. 12.d that neither zero vectors nor large vectors inject current in the neutral point N. So they do not change the voltage of neutral point.

Fig.12.b shows that medium vectors can inject current in neutral point if the load is unbalanced or nonlinear. Fig.12.c1 and Fig.12.c2 show two small vectors with two different switching combinations: (POO) and (ONN). These two vectors produce the same output voltage, but when the vector POO is applied, the current flows into the neutral point ($i_{NP} = -i_a$), while with the vector ONN, the current flows out ($i_{NP} = i_a$).

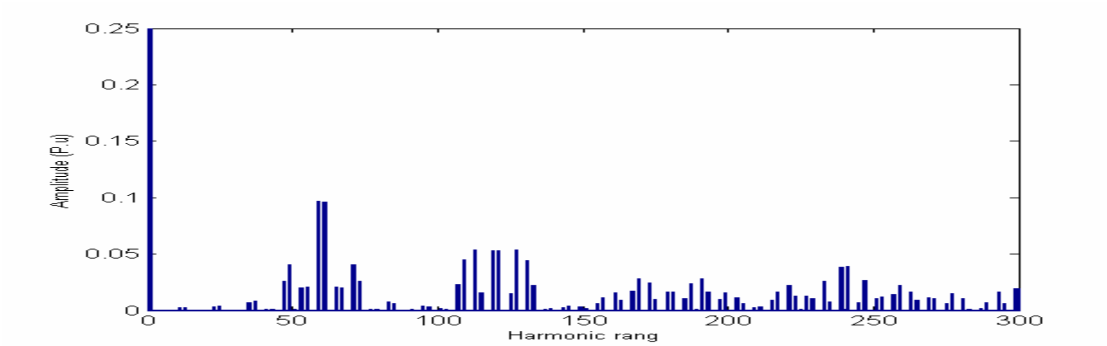
Table 5 shows the currents injected by all small and medium vectors [28]. As we see, each small redundant vector can inject either positive or negative current. Those small vectors injecting positive phase currents into the neutral point will be called positive vectors (ONN, PPO, NON, OPP, NNO, POP), while those injecting opposite phase currents will be called negatives vectors (POO, OON, OPO, NOO, OOP, ONO).

Medium vectors also affect neutral point potential. However, as they are not redundant vectors, this influence will not be controlled, being therefore considered as perturbation for the dc-voltage stabilization [8, 19].

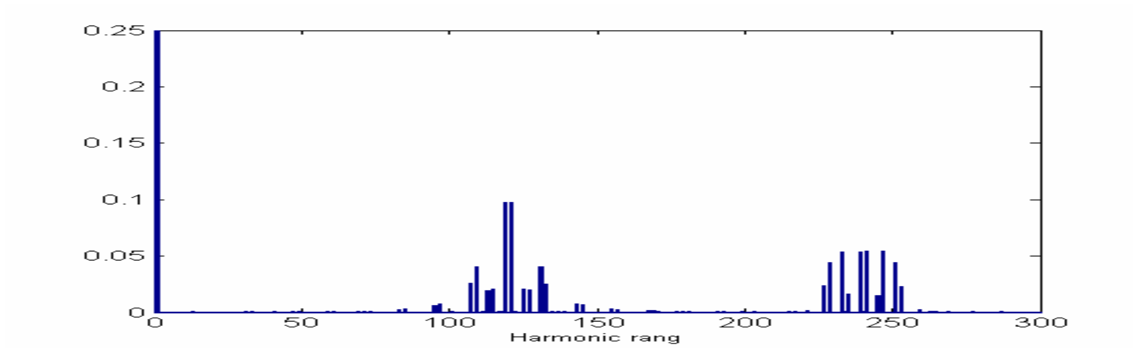
5.4 Neutral point potential control method

The model of the input dc-voltage of the inverter is given as [29]:

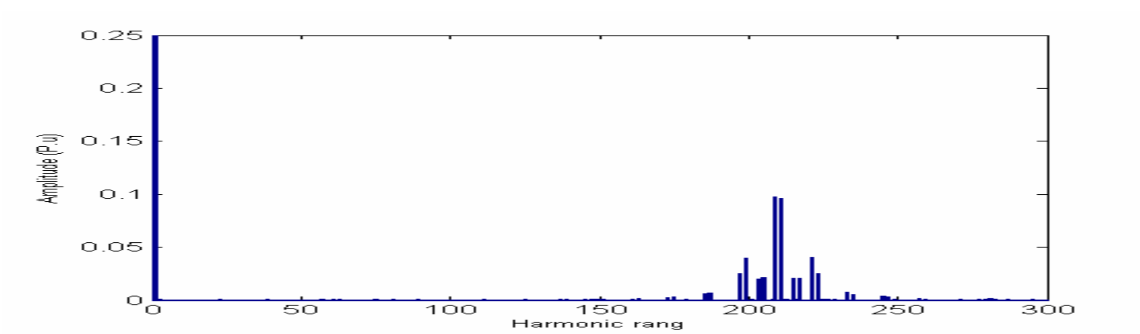
$$\begin{aligned}
&C\,\frac{dV_{dc1}}{dt} = I - i_{d1} \\
&C\,\frac{dV_{dc2}}{dt} = I + i_{d2} \\
&i_{NP} = -i_{d1} - i_{d2} \\
&i_{d1} = F_{11}.F_{12}.i_a + F_{21}.F_{22}.i_b + F_{31}.F_{32}.i_c \\
&i_{d2} = F_{13}.F_{14}.i_a + F_{23}F_{24}.i_b + F_{33}.F_{34}.i_c
\end{aligned} \tag{7}$$



a. n=60

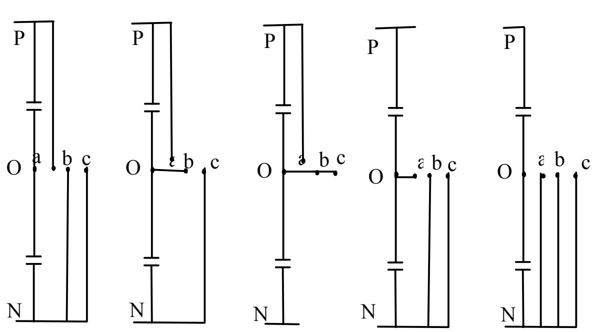


b. n=120



c. n=210

Figure 11. Effect of switching intervals number on spectrum of output voltage



a. PNN b. PON c_1. POO c_2. ONN d. NNN

Figure 12. Neutral point connections

Table 5. Neutral point current for different space vectors

Positive small vectors	i_{NP}	Negative small vectors	i_{NP}	Medium vectors	i_{NP}
ONN	i_a	POO	$-i_a$	PON	i_b
PPO	i_c	OON	$-i_c$	OPN	i_a
NON	i_b	OPO	$-i_b$	NPO	i_c
OPP	i_a	NOO	$-i_a$	NOP	i_b
NNO	i_c	OOP	$-i_c$	ONP	i_a
POP	i_b	ONO	$-i_b$	PNO	i_c

where V_{dc1} and V_{dc2} are capacitors voltages, i_{NP} is the neutral point current, i_{d1}, and i_{d2} are input currents of the inverter, and F_{ij} (i = 1,3, j = 1, 4) are switching signals of inverter switches, as indicated on Fig. 3.

The neutral point potential control is based on the use of both two redundant vectors in each sector, in order to inject positive or negative current in neutral point, depending on the value of the voltages of the two capacitors V_{dc1} and V_{dc2}, and the load current:

If we have $V_{dc1}>V_{dc2}$, in order to make $V_{dc1}= V_{dc2}$ we must inject a current either from O to P in order to reduce V_{dc1}, or from O to N in order to increase V_{dc2}. In both two cases, we have $i_{NP}< 0$.

If we have $V_{dc1}<V_{dc2}$, in order to make $V_{dc1}=V_{dc2}$, we must increase V_{dc1} by injecting a current from P to O, ore decrease V_{dc2} by injecting a current from N to O. In both two cases, we have $i_{NP} >0$.

Therefore, for insure the neutral point potential control we have to ensure the following relationship between V_{dc1}, V_{dc2} and i_{NP}:

$$(V_{dc1}-V_{dc2})\cdot i_{NP} <=0 \qquad (8)$$

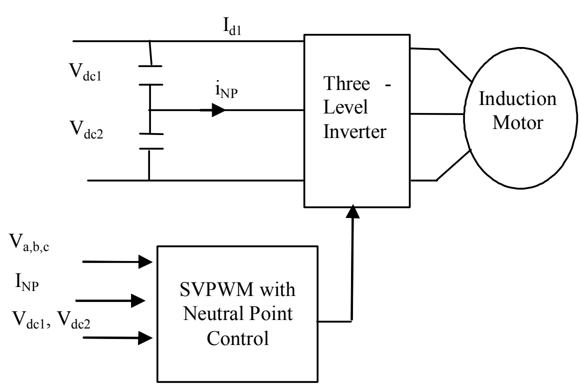


Figure 13. Scheme of Neutral Point Potential Control

Medium vectors influence will not be controlled, being therefore considered as a perturbation for the neutral point potential control. But if the load of the inverter is in balance, the currents injected by medium vectors will be cancelled mutually. In deed, for example the medium vectors PON and NOP inject the same current i_b as shown in Table 5, but there positions on the space vector diagram (Fig. 4) are symmetric regarding to the origin. Then their output voltages are equal and opposite in sign. So their phase currents are equal and opposite in sign, because the load is balanced.

The control scheme of the neutral point potential is given in Fig. 13. We make a continuous measurement of the two dc-voltages V_{dc1} and V_{dc2} and the neutral point current i_{NP} given by equation (8). We calculate the product $(V_{dc1} - V_{dc2})$. I_{NP}. If this product is negative we kept the set of small vectors (positive or negative one) used in the previous sample of time. Otherwise we must change this set of small vectors in the previous sample of time in order to inverse the sense of the neutral point current i_{NP}.

We simulate the association of dc-voltage input three-level inverter and induction motor, with introduction of the control loop. Simulation results on Fig. 14 show that before a brief transient period, the two dc-voltages being take equal and constant values (500V). The different between the two voltages tends to zero, even if we apply a load torque. This result prove the efficiency of the given control scheme which is simple but powerful.

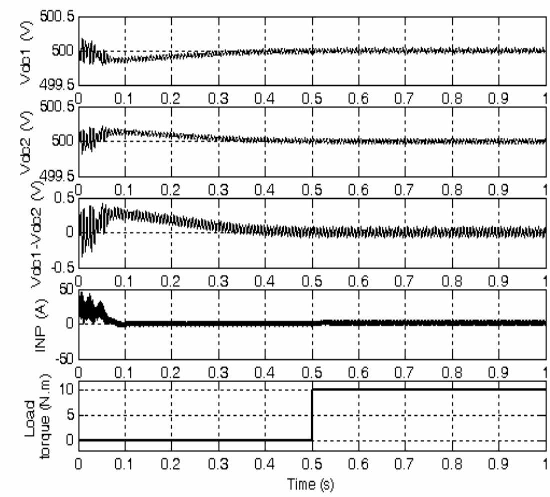


Figure 14. Controlled input voltages and current

6. CONCLUSIONS

In this paper, a simplified space vector pulse width modulation algorithm has been described and applied to three-level inverter. Through the decomposition of the space vector diagram the complicated three-level space vector modulation algorithm is simplified into two level cases. This simplified method has the following advantages.

- It reduces the execution time of the three-level inverter modulation
- It allows saving memory of the controller in case of experimental realization.
- The most important aspect of this algorithm lies in its generality. It can be used in any high-level inverter.

The neutral point potential can be easily controlled using redundant vectors and real time measurement of dc-voltages and neutral point current.

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Biographies

Djaafer LALILI was born in Jijel, Algeria, on 1973. He received the engineer degree and the master degree in Power Electronics from national polytechnic school of Algeria, in 1996 and 1999 respectively. Since 2000, he has been a lecturer in the Electrical Engineering department of Jijel University, Algeria. He is a researcher in electrical system study and modeling laboratory (LAMEL) at Jijel University. He is working in the field of power electronics, electrical drives and renewable energy.

El Madjid BERKOUK was born in Algeria, on 1968. He received his Engineering degree in 1991 in Electrical Engineering from the Polytechnic National School of Algiers, Algeria. In 1992 and 1995 he obtained, respectively, a Master of Science in ENSEEIHT (Toulouse) and Ph.D. in CNAM (Paris). From 1992 to 1996, he was teaching at the University of Paris XI. Since 1996, he is with the Polytechnic National school of Algiers as a Professor. His current research interests are power electronics and electrical drives.

Fares BOUDJEMA was born in Algiers, Algeria, on 1962. He received the Engineer degree in Electrical Engineering from the Ecole Nationale Polytechnique, Algiers, Algeria in 1985, the DEA degree, and the Doctorat degree in Automatic Control from the Université Paul Sabatier, Toulouse, France, in 1987, and 1991, respectively. In 1991, he joined the Department of Electrical Engineering, Ecole Nationale Polytechnique, Algiers, as an Assistant Professor. He was promoted to Associate Professor in 1994, and Professor in 2000. He has been the Director of the Laboratoire de Commande des Processus from July 2000 through March 2005. His research interests include application of sliding mode control, artificial neural network control, fuzzy control, and decentralized control in the field of the Electrical machines, power systems and robotics.

Nabil LOURCI was born in Jijel, Algeria, on 1976. He received the engineer degree and the master degree in Power Electronics from national polytechnic school of Algeria, in 1998 and 2000 respectively. Since 2001, he has been a lecturer in the Electrical Engineering department of Jijel University, Algeria. He is a researcher in electrical system study and modeling laboratory (LAMEL) at Jijel University. He is working in the field of power electronics, electrical drives and renewable energy.

ADAPTIVE CONTROL OF NONLINEAR SYSTEMS WITH UNKNOWN HIGH FREQUENCY GAIN AND DISTURBANCES

N. A. S. Fawzy [1,*], A. A. Abouelsoud [1], M. S. Mahmoud [2], R. I. Badr [1], M. F. Hassan [1]

[1] Electronics and Communications department, Faculty of Engineering, Cairo University, Giza, Egypt

[2] Graduate Studies, Cairo University, Postal Code 12311, Giza, Egypt

ABSTRACT

In this paper, the problem of designing a global robust model reference adaptive output feedback tracking control is presented for SISO nonlinear systems containing a vector of unknown constant parameters; entering linearly and subject to bounded disturbances with unknown bound. Furthermore, there is no a priori knowledge assumed on the sign of the high frequency gain. A Nussbaum gain is introduced in the global adaptive algorithm to ensure that the output tracks any bounded reference signal. Lyapunov stability method is applied and a dead zone criterion is used to guarantee that all the signals are globally bounded and the tracking error is arbitrary small.

Keywords

Nonlinear Systems, Bounded Disturbances, Nussbaum Gain, Tracking Control, Lyapunov Technique, Dead-Zone Criterion.

1. INTRODUCTION

Adaptive control theory has received a significant research attention in the past few years [7-9, 12, 13]. The problem of designing a global adaptive output-feedback tracking control for SISO nonlinear systems, with an unknown constant parameter vector entering linearly, was presented in [1]. Recently, the problem of designing a global robust adaptive control for SISO uncertain nonlinear systems having unknown differentiable time varying parameters and subject to bounded disturbances was solved in [2]. The previously mentioned control designs were proposed under the assumption that the sign of the high frequency gain is known. Thus, in [3, 14] a model reference adaptive control was presented for SISO nonlinear systems contained only unknown constant parameters that enter linearly in the state equations and were not subjected to disturbances. No a *priori* information on the sign of the high frequency gain was assumed. Hence, a new model reference adaptive tracking control algorithm with a Nussbaum gain [11] was proposed. Furthermore, in [4] a robust model reference adaptive control was designed for SISO linear dynamical system of unknown sign of high frequency gain and in the presence of disturbance of unknown bound. Recently, in [5] an adaptive control design of nonlinear systems with unknown high frequency gains which are perturbed by disturbances with unknown bounds was considered. However, the disadvantage of this technique is the over parameterization of the estimation of the unknown parameters. Moreover, there is burden in the implementation of the tuning functions due to the introduction of different orders of error terms in the Lyapunov functions. Moreover, in [9] an adaptive output regulator for nonlinear systems with unknown high frequency gain is proposed. Their results were limited due to regulation other than tracking and not including external disturbance with unknown band.

In this paper, an alternative technique different from that used in [5] is proposed to overcome the side effects of its control strategy. An adaptive control is developed for a certain class of nonlinear systems having unknown constant parameters and subject to unknown bounded disturbances. The main contribution of this paper is that no a *priori* information is assumed on the sign of the high frequency gain and the bounded disturbances are bounded by an unknown bound. A robust model reference adaptive output feedback control; with a Nussbaum gain; is designed that guarantees that the output tracks any bounded reference output. Lyapunov stability method is applied to guarantee the overall system stability. Furthermore, the dead zone technique presented in [6] is used to ensure the boundedness of all the signals. Finally, an example is given with simulation results to demonstrate the efficiency of the control strategy.

The paper is organized as follows. In Section 2, the problem statement is presented. Few notations and useful lemmas are stated in Section 3 along with the control design strategy. In Section 4, an example is given with simulation results to demonstrate the efficiency of the designed control. Finally, the conclusion is presented in Section 5.

2. PROBLEM STATEMENT

Consider the following class of SISO nonlinear systems with unknown constant parameters [3] and subject to bounded disturbances at the output:

$$\dot{x} = A_c x + a(\theta)y + b(\theta)\,\sigma(y)\,u + \psi_o(y) + \sum_{i=1}^{p} \psi_i(y)\,\theta_i$$
$$y = C_c x + v(t), \;\; x \in R^n, u \in R, \theta \in R^p, \, y \in R \tag{1}$$

where x is the state vector, u is the control, y is the output, $\theta = [\theta_1, \ldots, \theta_p]^T$ is the constant unknown parameter vector, the vector $b(\theta)$ is Hurwitz with relative degree ρ for every θ,i.e., $b_\rho(\theta) \neq 0$, where $b_\rho(\theta)$ is the high frequency gain of

*Corresponding author: *E-mail:* magdim@yahoo.com

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the system, and $v(t)$ is a bounded disturbance with unknown bound and has bounded time derivative.

where $0 \neq \sigma: R \rightarrow R$, $\psi_i: R \rightarrow R^n$, $0 \leq i \leq p$ and (A_c, b, C_c) are in observer canonical form:

$$A_c = \begin{bmatrix} 0 & I_{n-1\times n-1} \\ 0 & 0 \end{bmatrix}, b(\theta) = [0 \ldots 0\ b_\rho(\theta) \ldots b_n(\theta)]^T,\ C_c = [1\ 0\ 0\ \ldots 0] \tag{2}$$

$$a(\theta) = [a_1(\theta) \ldots a_{\rho-1}(\theta) 0, \ldots 0]^T,\ \psi_i(y) = [0 \ldots 0\ \psi_{i,\rho}(y) \ldots \psi_{i,n}(y)]$$

By rearranging the nonlinear terms, (1) can take the following form [3]:

$$\dot{x} = A_c x + a(\theta) y + b(\theta)\,\sigma(y)\,u + \sum_{i=1}^{p'} d'_i \phi'_i(y) + \sum_{i=1}^{p''} d''_i \phi''_i(y)$$
$$y = C_c x + v(t) \tag{3}$$

where p' and p'' denote the numbers of unique nonlinear elements in ψ_0 and in $\sum_{i=1}^{p} \psi_i(y)\theta_i$, respectively, $(\phi'_i(y)): R \rightarrow R^n$, $1 \leq i \leq p'$ and $(\phi''_i(y)): R \rightarrow R^n$, $1 \leq i \leq p''$ denote those nonlinear terms, $d'_i \in R^n$, $1 \leq i \leq p'$ are known constant vectors and $d''_i \in R^n$, $1 \leq i \leq p''$ are constant vectors depending on θ. From the special form of $\psi_i(y)$, we can conclude that:

$$d'_{i,j} = 0, 1 \leq i \leq p', d''_{i,j} = 0, 1 \leq i \leq p'', 1 \leq j \leq \rho - 1 \tag{4}$$

Define the following reference model:

$$W_m(s) = k_m \frac{Z_m(s)}{R_m(s)} \tag{5}$$

which is minimum phase, stable and with relative degree ρ, $R_m(s)$ and are monic, Hurwitz polynomials with orders n and n-ρ, respectively and k_m is the high frequency gain. Thus, the reference output is:

$$y_m = W_m\, r \tag{6}$$

where r is a bounded reference input. Define the following transfer matrix [3]:

$$W(s) = \frac{\alpha(s)}{Z_m(s) P(s)} \tag{7}$$

where $P(s)$ is monic, Hurwitz polynomial with order ρ-1, and:

$$\alpha(s) = [s^{n-2} \quad s^{n-3} \ldots 1]^T \text{ for } n \geq 2 \text{ and } 0 \text{ for } \quad n = 1 \tag{8}$$

Theorem 2 presented in [3] is now modified to include the additional term $v(t)$ representing the bounded disturbance.

Theorem 1: There exists a $\bar{\theta}$ such that system (3) can be expressed as:

$$y = W_m\, k\, (\sigma(y) u - \bar{\theta}^T \bar{\omega}) + v_1(t) \tag{9}$$

where $k = b_\rho / k_m$ and:

$$\bar{\omega} = [W^T \sigma(y) u \quad W^T y \quad y \quad \omega'^T \quad \omega''^T]^T \tag{10}$$

$$\bar{\theta} = [\theta_u^T \quad \theta_y^T \quad \theta_{y,0} \quad \theta'^T \quad \theta''^T]^T \tag{11}$$

with:

$$\omega' = [W^T \phi'_1(y) \quad \phi'_1(y) \ \ldots \quad W^T \phi'_{p'}(y) \quad \phi'_{p'}(y)]^T \tag{12}$$

$$\theta' = [\theta'^T_1 \quad \theta'_{1,0} \ \ldots \quad \theta'^T_{p'} \quad \theta'_{p',0}]^T \tag{13}$$

$$v_1(s) = \frac{R_p(s) Q(s)}{R_m(s) P(s)}\, v(s) \tag{14}$$

where ω'' and θ'' have the same format of ω' and θ' respectively and $v_1(t)$ is the effect of the disturbance on the output.

Proof:

Assuming zero initial conditions, system (1) can take the following form:

$$y(s) = b_\rho \left(\frac{Z_p(s)}{R_p(s)} L\{\sigma(y) u\} + \sum_{i=1}^{p'} \frac{Z'_i(s)}{R_p(s)} L\{\phi'_i(y)\} + \sum_{i=1}^{p''} \frac{Z''_i(s)}{R_p(s)} L\{\phi''_i(y)\} \right) + v(t) \tag{15}$$

where $L\{f(t)\}$ is the Laplace transform of $f(t)$ and:

$$b_\rho \frac{Z_p(s)}{R_p(s)} = C_c (sI - (A_C + aC_c))^{-1} b \tag{16}$$

$$b_\rho \frac{Z'_i(s)}{R_p(s)} = C_c (sI - (A_C + aC_c))^{-1} d'_i, \qquad 1 \leq i \leq p' \tag{17}$$

$$b_\rho \frac{Z''_i(s)}{R_p(s)} = C_c (sI - (A_C + aC_c))^{-1} d''_i, \qquad 1 \leq i \leq p'' \tag{18}$$

with $R_p(s)$ monic and of order n, $Z_p(s)$ monic and $Z_p(s)$, $Z'_i(s)$, $1 \leq i \leq p'$, $Z''_i(s)$, $1 \leq i \leq p''$ of order n-ρ. Moreover, consider the following identity:

$$R_p(s)Q(s) - b_\rho\left(\theta_y^T\alpha(s) + \theta_{y,0}Z_m(s)P(s)\right) = R_m(s)P(s) \tag{19}$$

where $Q(S)$ is monic and of order ρ-1. There exist $Q(S)$, θ_y, $\theta_{y,0}$ for given $R_p(s)$, $R_m(s)$ to satisfy the above identity. It follows that for a given θ, there exist θ'_u, θ'_i, $\theta'_{i,0}$, $1 \le i \le p'$, θ''_i, $\theta''_{i,0}$, $1 \le i \le p''$ satisfying:

$$\theta_u^T\alpha(s) = Z_m(s)P(s) - Z_p(s)Q(s) \tag{20}$$

$$\theta_i'^T\alpha(s) + \theta'_{i,0}Z_m(s)P(s) = -Q(s)Z'_i(s), \qquad 1 \le i \le p' \tag{21}$$

$$\theta_i''^T\alpha(s) + \theta''_{i,0}Z_m(s)P(s) = -Q(s)Z''_i(s), \qquad 1 \le i \le p'' \tag{22}$$

Thus, we can get identity (9) by manipulating identity (15) with (19)-(22) as follows.

From (20), we get:

$$\left(\theta_u^T\frac{\alpha(s)}{Z_m(s)P(s)}\right)\sigma(y)u = \left(1 - \frac{Z_p(s)Q(s)}{Z_m(s)P(s)}\right)\sigma(y)u$$

$$\left(\theta_u^TW(s)\right)\sigma(y)u = \left(1 - \frac{Z_p(s)Q(s)}{Z_m(s)P(s)}\right)\sigma(y)u \tag{23}$$

From (21), we have:

$$\left(\theta_i'^T\frac{\alpha(s)}{Z_m(s)P(s)}\right)\phi'_i(y) + \theta'_{i,0}\phi'_i(y) = -\left(\frac{Q(s)Z'_i(s)}{Z_m(s)P(s)}\right)\phi'_i(y), \quad 1 \le i \le p'$$

$$\left(\theta_i'^TW(s)\right)\phi'_i(y) + \theta'_{i,0}\phi'_i(y) = -\left(\frac{Q(s)Z'_i(s)}{Z_m(s)P(s)}\right)\phi'_i(y), \quad 1 \le i \le p' \tag{24}$$

Similarly:

$$\left(\theta_i''^TW(s)\right)\phi''_i(y) + \theta''_{i,0}\phi''_i(y) = -\left(\frac{Q(s)Z''_i(s)}{Z_m(s)P(s)}\right)\phi''_i(y), \quad 1 \le i \le p'' \tag{25}$$

From (19), we can get this form:

$$\frac{R_p(s)Q(s)}{Z_m(s)P(s)}y - b_\rho\left(\theta_y^TW(s) + \theta_{y,0}\right)y = \frac{R_m(s)}{Z_m(s)}y \tag{26}$$

From (23)-(26) and (10)-(13), we obtain:

$$\begin{aligned}\bar\theta^T\bar\varpi &= \left(1 - \frac{Z_p(s)Q(s)}{Z_m(s)P(s)}\right)\sigma(y)u - \left(\frac{Q(s)Z'_i(s)}{Z_m(s)P(s)}\right)\phi'_i(y) \\ &- \left(\frac{Q(s)Z''_i(s)}{Z_m(s)P(s)}\right)\phi''_i(y) + \frac{R_p(s)Q(s)}{b_\rho Z_m(s)P(s)}y - \frac{R_m(s)}{b_\rho Z_m(s)}y \\ &= \sigma(y)u + \frac{Q(s)}{Z_m(s)P(s)}\Bigg(-Z_p(s)\sigma(y)u - Z'_i(s)\phi'_i(y) \\ &- Z''_i(s)\phi''_i(y) + \frac{R_p(s)y}{b_\rho}\Bigg) - \frac{R_m(s)}{b_\rho Z_m(s)}y\end{aligned} \tag{27}$$

From (15), we get:

$$\bar\theta^T\bar\varpi = \sigma(y)u - \frac{R_m(s)}{b_\rho Z_m(s)}y + \frac{R_p(s)v}{b_\rho} \tag{28}$$

By rearranging the terms, (9) can be obtained and the Theorem is proved. Furthermore, define the measured tracking error $r = y - y_m$, hence:

$$e = W_m\, k\,(\sigma(y)u - \theta^T\omega) + v_1(t) \tag{29}$$

where:

$$\theta = \left[\bar\theta^T, 1/k\right]^T, \qquad \omega = \left[\bar\varpi, r\right]^T \tag{30}$$

From (15), we can conclude that the control takes this form:

$$u = \sigma^{-1}(y)\hat\theta^T\omega \tag{31}$$

where $\hat\theta$ is the estimate of θ. Hence, (15) becomes:

$$e = W_m\, k\,(\tilde\theta^T\omega) + v_1(t) \tag{32}$$

where $\tilde\theta = \hat\theta - \theta$. Since the transfer function W_m is known, we can generate an auxiliary error signal e_2 as in [6], [7], and [8]:

$$e_2 = \hat\theta^TW_m\omega - W_m\hat\theta^T\omega \tag{33}$$

Moreover, define:

$$\zeta = W_m\omega \tag{34}$$

Thus, (19) can take this form

$$e_2 = \tilde{\theta}^T \zeta - W_m \tilde{\theta}^T \omega \tag{35}$$

In the following section, the tracking problem is solved with the following growth conditions (where c is a generic notation for constant positive real):

$$|\phi_i'(y)| \le c\,|y| + c,\ 1 \le i \le p',\ |\phi_i''(y)| \le c\,|y| + c,\ 1 \le i \le p'' \tag{36}$$

3. MAIN RESULT

Few notations and useful lemmas are introduced in order to prove the boundedness of the signals in the system. The exponential weighted norm in L_2 with $\delta > 0$ is defined by:

$$\|x_t\|_{2\delta} = \left(\int_0^t e^{-\delta(t-\tau)} x^T(\tau)\, x(\tau)\, d\tau \right)^{\frac{1}{2}}$$

The following lemma is associated with the input-output properties in $\|\cdot\|_{2\delta}$.

Lemma 1 [3]:

Let $y = H(s)u$. If $H(s)$ is proper and analytic in $R[s] \ge -\delta/2$ for some $\delta \ge 0$ then:

(i) $\|y_t\|_{2\delta} \le \|H(s)\|_{\infty\delta}\, \|u_t\|_{2\delta}$ with $\|H(s)\|_{\infty\delta} = \sup_\omega |H(j\omega - \delta/2)|$

(ii) Furthermore, when $H(s)$ is strictly proper, we have:

$$|y(t)| \le \|H(s)\|_{2\delta}\, \|u_t\|_{2\delta}$$

where, for any $p > \delta/2 \ge 0$

$$\|H(s)\|_{2\delta} = \frac{1}{\sqrt{2\pi}} \left(\int_{-\infty}^{\infty} \left(H(j\omega - \delta/2)\right)^2 d\omega \right)^{\frac{1}{2}} \le \frac{1}{\sqrt{2p-\delta}} \|\,(s+p)\,H(s)\|_{\infty\delta}$$

Two swapping lemmas are also presented.

Lemma 2 (Swapping Lemma 1) [3]:

Let $\tilde{\theta}$, $\omega{:}R^+ \to R^m$ and $\tilde{\theta}$ be differentiable. Let $G(s)$ be a proper stable rational transfer function with a minimal realization (A, B, C, D), that is $G(s) = C^T(sI - A)^{-1}B + d$. Then

$$G(s)\tilde{\theta}^T \omega = \tilde{\theta}^T G(s)\omega - G_c(s)\left(\left(G_b(s)\omega^T\right)\dot{\tilde{\theta}}\right)$$

where:

$$G_c(s) = C^T\left(sI - A\right)^{-1}, \quad G_b(s) = \left(sI - A\right)^{-1} B$$

Lemma 3 (Swapping Lemma 2) [3]:

Let $\tilde{\theta}$, $\omega{:}R^+ \to R^m$ and $\tilde{\theta}$, ω be differentiable. Then:

$$\tilde{\theta}^T \omega = F_1(s,\alpha_o)\left(\dot{\tilde{\theta}}^T \omega + \tilde{\theta}^T \dot{\omega}\right) + F(s,\alpha_o)\left(\tilde{\theta}^T \omega\right)$$

where:

$$F(s,\alpha_o) = \alpha_o{}^{\rho} / \left(s + \alpha_o\right)^{\rho},$$
$$F_1(s,\alpha_o) = \left(1 - F(s,\alpha_o)\right)/\,s,\ \rho \ge 1 \text{ and } \alpha_o$$

arbitrary positive real. Furthermore, if $\alpha_0 > \delta \ge 0$, then:

$$\|F_1(s,\alpha_o)\|_{\infty\delta} \le \frac{c}{\alpha_o}$$

where c is a finite positive real independent of α_0. For convenience, we will use $\|\cdot\|$ to denote $\|(\cdot)_t\|_{2\delta}$ with $\delta > 0$ and c for any bounded positive real.

In the theorem to follow, a model reference adaptive tracking control is proposed for the case of nonlinear systems with unknown sign of the high frequency gain and subjected to bounded disturbances with unknown bound. Lyapunov technique is applied and the dead zone criterion is used to guarantee the boundedness of all the signals in the system.

Theorem 2:

A global robust model reference adaptive tracking control (17) exists for the nonlinear systems (1), satisfying equation (22), having unknown constant parameters entering linearly and the sign of the high frequency gain is unknown. Furthermore, the nonlinear systems are subjected to bounded disturbances with unknown bound.

Proof: Define the augmented error e_1:

$$e_1 = e + N(x)\,\hat{k}\,\,e_2 \tag{37}$$

where $\hat{k}$ is the estimate of k and $N(x)$ is the Nussbaum gain as follows:

$$N(x) = x^2 \cos(x) \tag{38}$$

where x is defined later on. From (32) and (35), (37) takes the following form:

$$e_1 = k\tilde{\theta}^T \zeta - ke_2 + N(x)\,\hat{k}\,\,e_2 + v_1 \tag{39}$$

Define the Lyapunov function:

$$V = \frac{1}{2}\,\tilde{\theta}^T \Gamma^{-1} \tilde{\theta} + \frac{1}{2\alpha}\,\frac{k_m}{|b_\rho|}\,\tilde{\beta}^2 \tag{40}$$

where Γ is a symmetric positive definite matrix, α is a positive constant, β is the unknown upper bound of v_1 such that $\beta \ge |v_1|$, $\hat{\beta}$ is the estimate of β and $\tilde{\beta} = \hat{\beta} - \beta$.

By differentiating (40), we get:

$$\dot{V} = \tilde{\theta}^T \Gamma^{-1} \dot{\tilde{\theta}} + \frac{1}{\alpha} \frac{k_m}{|b_\rho|} \tilde{\beta}\, \dot{\tilde{\beta}} \tag{41}$$

Therefore, with reference to [4], define the following adaptations:

$$\dot{\tilde{\theta}} = N(x) \frac{\Gamma \eta\, e_1\, \zeta}{(1+\zeta^T\zeta)(1+N^2(x))} \tag{42}$$

where η is the dead zone function depending on $\hat{\beta}$ as follows:

$$\eta = 1 \quad if\ \hat{\beta}\, x^2 < |e_1| \quad and \quad \eta = 0 \quad otherwise \tag{43}$$

$$\dot{\tilde{\beta}} = |N(x)| \frac{\alpha \eta\, |e_1|}{(1+\zeta^T\zeta)(1+N^2(x))} \tag{44}$$

$$\dot{\hat{k}} = -N(x) \frac{\eta\, e_1\, e_2}{(1+\zeta^T\zeta)(1+N^2(x))} \tag{45}$$

$$\dot{z} = \frac{\eta\, e_1^2}{(1+\zeta^T\zeta)(1+N^2(x))} \tag{46}$$

$$x = \mathrm{z} + \frac{\hat{\mathrm{k}}^2}{2} \tag{47}$$

By substituting (42) and (44) in (51) and by using (39), we get:

$$\begin{aligned}\dot{V} = {} & N(x) \frac{k_m}{b_\rho} \left(\frac{\eta\, e_1^2 - N(x) \hat{k} \eta\, e_1\, e_2}{(1+\zeta^T\zeta)(1+N^2(x))} \right) \\ & + N(x) \frac{\eta\, e_1\, e_2}{(1+\zeta^T\zeta)(1+N^2(x))} \\ & - N(x) \frac{k_m}{b_\rho} \left(\frac{\eta\, e_1\, v_1}{(1+\zeta^T\zeta)(1+N^2(x))} \right) \\ & + \frac{k_m}{|b_\rho|} |N(x)| \frac{\eta\, |e_1| \left(\hat{\beta} - \beta\right)}{(1+\zeta^T\zeta)(1+N^2(x))}\end{aligned} \tag{48}$$

From (45) and (46), we have:

$$\begin{aligned}\dot{V} \le {} & N(x) \frac{k_m}{b_\rho} \left(\dot{z} + \hat{k}\, \dot{\hat{k}} \right) - \dot{\hat{k}} \\ & + \frac{k_m}{|b_\rho|} |N(x)| \frac{\eta\, |e_1|\, \hat{\beta}}{(1+\zeta^T\zeta)(1+N^2(x))} \\ & + \frac{k_m}{|b_\rho|} |N(x)| \frac{\eta\, |e_1| \left(|v_1| - \beta\right)}{(1+\zeta^T\zeta)(1+N^2(x))}\end{aligned} \tag{49}$$

From (43), (46) and (47), we get:

$$\dot{V} \le N(x) \frac{k_m}{b_\rho} \left(\dot{z} + \hat{k}\, \dot{\hat{k}} \right) - \dot{\hat{k}} + \frac{k_m}{|b_\rho|} \frac{\eta\, e_1^2}{(1+\zeta^T\zeta)(1+N^2(x))} \tag{50}$$

Hence:

$$\dot{V} \le N(x) \frac{k_m}{b_\rho} \dot{x} - \dot{\hat{k}} + \frac{k_m}{|b_\rho|} \dot{z} \tag{51}$$

By integrating both sides, we get:

$$\begin{aligned}V(t) \le {} & \frac{k_m}{b_\rho} \int_{x(t_0)}^{x(t)} N(x)\, dx - \left(\hat{k}(t) - \hat{k}(t_0) \right) \\ & + \frac{k_m}{|b_\rho|} z(t) - \frac{k_m}{|b_\rho|} z(t_0) + \frac{1}{2} \tilde{\theta}^T(t_0) \Gamma^{-1} \tilde{\theta}(t_0) \\ & + \frac{1}{2\alpha} \frac{k_m}{|b_\rho|} \tilde{\beta}^2(t_0)\end{aligned} \tag{52}$$

Since

$$2\left|\hat{k}\right| \le \left|\hat{k}\right|^2 \frac{k_m}{|b_\rho|} + \frac{|b_\rho|}{k_m} \tag{53}$$

Therefore:

$$\begin{aligned}V(t) \le {} & \frac{k_m}{b_\rho} \int_{x(t_0)}^{x(t)} N(x)\, dx + \frac{k_m}{|b_\rho|} x(t) + \frac{|b_\rho|}{2k_m} \\ & + \hat{k}(t_0) - \frac{k_m}{|b_\rho|} z(t_0) + \frac{1}{2} \tilde{\theta}^T(t_0) \Gamma^{-1} \tilde{\theta}(t_0) \\ & + \frac{1}{2\alpha} \frac{k_m}{|b_\rho|} \tilde{\beta}^2(t_0)\end{aligned} \tag{54}$$

which can be presented in the following form:

$$V(t) \leq f(x) + f_1 \tag{55}$$

where:

$$f(x) = \frac{k_m}{b_\rho}\left(x^2 \sin(x) + 2x\cos(x) - 2\sin(x)\right) + \frac{k_m}{|b_\rho|}x \tag{56}$$

$$f_1 = \frac{|b_\rho|}{2k_m} + \hat{k}(t_0) - \frac{k_m}{|b_\rho|}z(t_0) + \frac{1}{2}\tilde{\theta}^T(t_0)\Gamma^{-1}\tilde{\theta}(t_0) + \frac{1}{2\alpha}\frac{k_m}{|b_\rho|}\tilde{\beta}^2(t_0) + f(x(t_0)) \tag{57}$$

From (56), if $x(t)$ is unbounded, then there exists an interval in x that makes $f(x)$ highly negative consequently; the right hand side of (55) becomes negative. Therefore, there is a contradiction as the left hand side of (55) is positive. Hence, x is bounded and consequently, N, V, $\hat{k}$, $\hat{\beta}$, z and $\tilde{\theta} \in L_\infty$. Moreover, from (42) and (46) we conclude that $\dot{\tilde{\theta}}$ *and* $e_1/\sqrt{(1+\zeta^T\zeta)} \in L_2$.

In order to prove that all signals are bounded, define [3]:

$$m^2 = 1 + \|\sigma(y)u\|^2 + \|y\|^2 \tag{58}$$

From equations (19) and (21), we get:

$$y = W_m\ (r + k\tilde{\theta}^T\omega) + v_1(t) \tag{59}$$

Since W_m is strictly proper, and from Lemma 1, we get:

$$\|y\| \leq c + c\left\|\tilde{\theta}^T\omega\right\| \tag{60}$$

$$|y| \leq c + c\left\|\tilde{\theta}^T\omega\right\| \tag{61}$$

From (8), we have:

$$\sigma(y)u = \frac{1}{b_\rho}\frac{R_p(s)}{Z_p(s)}\left(W_m\ (r + k\tilde{\theta}^T\omega) + v_1(t) - v(t)\right) - \sum_{i=1}^{p'}\frac{Z_i'(s)}{Z_p(s)}\{\phi_i'(y)\} - \sum_{i=1}^{p''}\frac{Z_i''(s)}{Z_p(s)}\{\phi_i''(y)\} \tag{62}$$

Since

$$\left(R_p/Z_p\right)W_m, Z_i'/Z_p, 1 \leq i \leq p' \text{ and } Z_i''/Z_p, 1 \leq i \leq p''$$

are proper and stable, the growth conditions of (36) and

$$\left(R_p/b_\rho Z_p\right)\left(v_1(t)/v(t) - 1\right) = \left(R_p/R_m P Z_p\right)\left(\theta_y^T\alpha + \theta_{y,0}Z_m P\right)$$

is strictly proper, hence from Lemma 1, we obtain:

$$\|\sigma(y)u\| \leq c + c\left\|\tilde{\theta}^T\omega\right\| \tag{63}$$

It follows from (58), (60) and (63), that

$$m^2 \leq c + c\left\|\tilde{\theta}^T\omega\right\|^2 \tag{64}$$

And, we prove that the signals are bounded by m. Since every transfer function in W is proper, from equations (10) and (12) and Lemma 1, we get $\|\omega\| \leq c + cm$. Since $\tilde{\theta} \in L_\infty$ and from (61), we obtain $|y| \leq c + cm$. Hence, from (36), and since W is strictly proper, we get $|y| \leq c + cm$. Consequently, from (31), we conclude that $|\sigma(y)u| \leq c + cm$. Moreover, since W_m is strictly proper and from (48), we have $|\zeta| \leq c + cm$. Furthermore, from equation (59) we get $\dot{y} = sW_m(r + k\tilde{\theta}^T\omega) + sv_1$ and since sW_m are proper and the derivative of v_1 are bounded, therefore:

$$\|\dot{y}\| \leq c + c\,\|\omega\| \leq c + c\,m \tag{65}$$

Similarly, we can prove that $|\dot{\omega}| \leq c + cm$. Therefore, we conclude that:

$$\frac{|y|}{m}, \frac{|\sigma(y)u|}{m}, \frac{|\omega|}{m}, \frac{|\zeta|}{m}, \frac{|\dot{y}|}{m}, \frac{|\dot{\omega}|}{m} \in L_\infty \tag{66}$$

By applying Lemma 2, with $G(s) = W_m$ in (35), we get:

$$e_2 = G_c(s)\left(\left(G_b(s)\omega^T\right)\dot{\tilde{\theta}}\right) \tag{67}$$

Thus, from (66) and (67) and from Lemma 1, we conclude that:

$$\|e_2\| \leq c\left\|\dot{\tilde{\theta}}\, m\right\| \tag{68}$$

Moreover, from (35) and (39):

$$e_1 = kW_m\,\tilde{\theta}^T\omega + N(x)\,\hat{k}\,e_2 + v_1 \tag{69}$$

Hence:

$$\tilde{\theta}^T\omega = \frac{1}{k}W_m^{-1}\left(e_1 - N(x)\,\hat{k}\,e_2 - v_1\right) \tag{70}$$

By applying Lemma 3, we get:

$$\tilde{\theta}^T\omega = F_1\left(\dot{\tilde{\theta}}^T\omega + \tilde{\theta}^T\dot{\omega}\right) + \frac{F}{k}W_m^{-1}\left(e_1 - N(x)\,\hat{k}\,e_2 - v_1\right) \tag{71}$$

Since FW_m^{-1} is proper and by applying Lemma 3, we obtain:

$$\left\|FW_m^{-1}\right\|_{\infty\delta} \le c\alpha_o^{\rho}, \quad \left\|F_1\right\|_{\infty\delta} \le \frac{c}{\alpha_o} \tag{72}$$

By taking 2δ norm of equation (71) and by applying Lemma 1, we get:

$$\left\|\tilde{\theta}^T\omega\right\| \le \frac{c}{\alpha_o}\left(\left\|\dot{\tilde{\theta}}^T\omega\right\| + \left\|\tilde{\theta}^T\dot{\omega}\right\|\right) + c\alpha_o^{\rho}\left(\|e_1\| + \|e_2\| + \|v_1\|\right) \tag{73}$$

where $\|v_1\|$ is bounded. Since $e_1/\sqrt{(1+\zeta^T\zeta)} \in L_2$, let $e_1 = \gamma\sqrt{(1+\zeta^T\zeta)}$ where $\gamma \in L_2$. Thus,

$$\|e_1\| < \gamma\sqrt{\left(1+2|\zeta| + |\zeta|^T|\zeta|\right)} < \gamma|\zeta|$$

Therefore, we conclude that:

$$\left\|\dot{\tilde{\theta}}^T\omega\right\| \le c\left\|\dot{\tilde{\theta}}\, m\right\|, \quad \left\|\tilde{\theta}^T\dot{\omega}\right\| \le c\, m, \|e_1\| < c\|\gamma\, m\|, \; \|e_2\| \le c\left\|\dot{\tilde{\theta}}\, m\right\| \tag{74}$$

By substituting (74) in (75), we obtain:

$$\begin{aligned}\left\|\tilde{\theta}^T\omega\right\| &\le \frac{c}{\alpha_o}m + c\alpha_o^{\rho}\left(\alpha_o^{-\rho-1}\left\|\dot{\tilde{\theta}}\, m\right\| + \|\gamma\, m\| + \right| \\ &\le \frac{c}{\alpha_o}m + c\alpha_o^{\rho}\|g\, m\|\end{aligned} \tag{75}$$

where $\alpha_0 > 1$, $g^2 = |\dot{\tilde{\theta}}|^2 + \gamma^2$ and $g \in L_2$. Substitute (75) in (64):

$$m^2 \le c + \frac{c}{\alpha_o^2}m^2 + c\alpha_o^{2\rho}\|g\, m\|^2 \tag{76}$$

For a large enough α_0, $m^2 \le c + \frac{c}{\alpha_0^2}m^2 + c\alpha_0^{2\rho}\|gm\|^2$ which takes the following form:

$$m^2 \le c + c\int_0^t e^{-\delta(t-\tau)}g^2(\tau)\,m^2(\tau)\,d\tau \tag{77}$$

By applying Bellman - Gronwall Lemma [10], we have:

$$m^2 \le c + c\int_0^t e^{-\delta(t-\tau)}g^2(\tau)\,e^{\int_\tau^t g^2(\mu)d\mu}\,d\tau \tag{78}$$

Therefore, the boundedness of m and $m \in L_\infty$, is concluded from $g \in L_2$. The boundedness of the other signals is deduced from equation (66). Hence, y is bounded and since y_m is bounded, therefore the tracking error e is also bounded. Consequently, all the signals are bounded and the Theorem is proved.

4. EXAMPLE AND SIMULATION RESULTS

Consider the following nonlinear system [3] with an additional term representing the bounded disturbance with unknown bound:

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= \theta_1 u + \theta_2\cos y\\ y &= x_1 + v(t)\end{aligned}$$

This example is of relative degree 2 with the following reference model [3]:

$$W_m(s) = \frac{1}{(s+1)(s+2)}$$

And the filter $W(s)$ is of first order and can take it as $W(s) = \frac{1}{s+1}$. Furthermore, we have:

$$\begin{aligned}\theta &= \left(-3, \frac{3}{\theta_1}, -\frac{5}{\theta_1}, -\frac{3\theta_2}{\theta_1}, -\frac{\theta_2}{\theta_1}, \frac{1}{\theta_1}\right),\\ \omega &= \left(\frac{u}{s+1}, \frac{y}{s+1}, y, \frac{\cos y}{s+1}, \cos y, r\right),\\ k &= \theta_1\end{aligned}$$

Fig. 1 and 2 represent the simulation results corresponding to the following initial conditions, parameters and gain matrices:

$$\begin{aligned}&x(0) = [0, -1]^T, \theta_1 = \theta_2 = 1, \hat{\theta}_1(0) = \hat{\theta}_2(0) = 0, \hat{k}(0) = 1,\\ &r = 60\sin(0.2\,t), v = 10\sin(0.02\,t), \alpha = 0.5, \Gamma = 10.\end{aligned}$$

Fig. 1 show clearly that the output of the system tracks the reference output and that the error between them decreases with time in Fig. 2 thus demonstrating the efficiency of the robust model reference adaptive output feedback tracking control shown in Fig. 3.

To prove that the adaptive control is global, simulation results Fig. 4 - Fig. 6 are given corresponding to different initial conditions $x(o) = [1, 1]^T$, $\alpha = 1$

It is readily from Fig. 4-6 that the output has succeeded to track the reference output and that the robust adaptive tracking control is global.

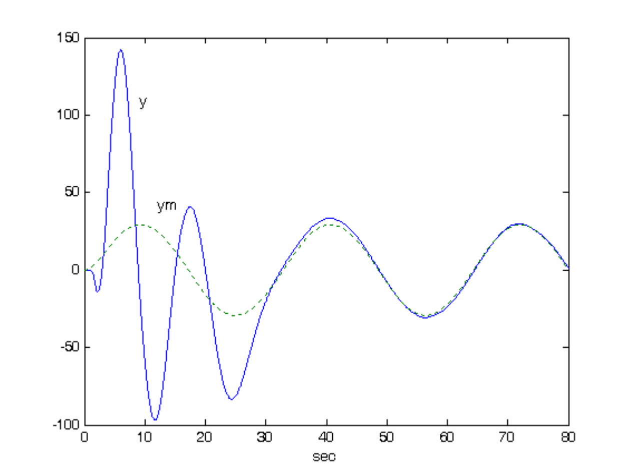

Figure 1. The reference output y_m and the output of the system y

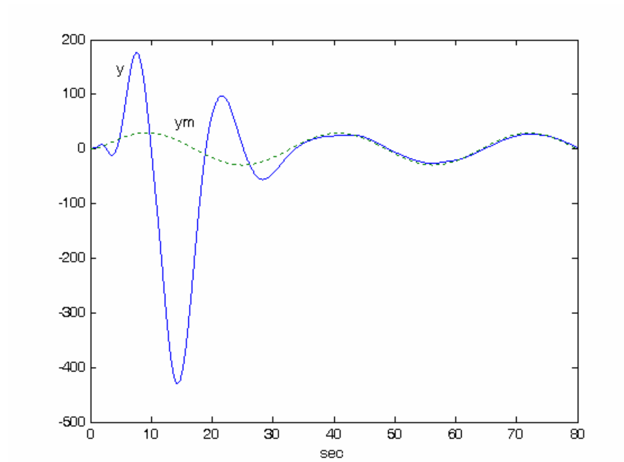

Figure 4. The reference output y_m and the output of the system y

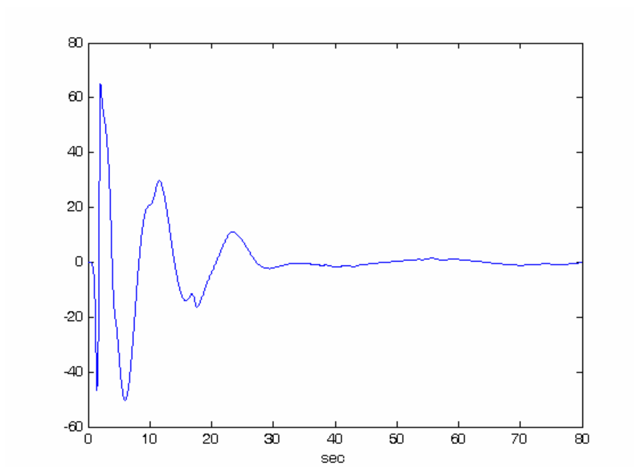

Figure 2. The model reference adaptive tracking control

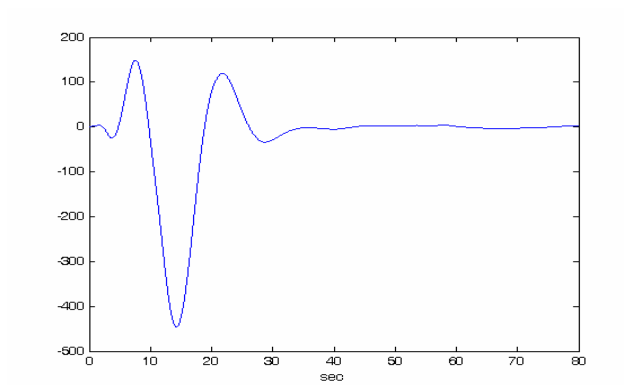

Figure 5. the error between y and y_m

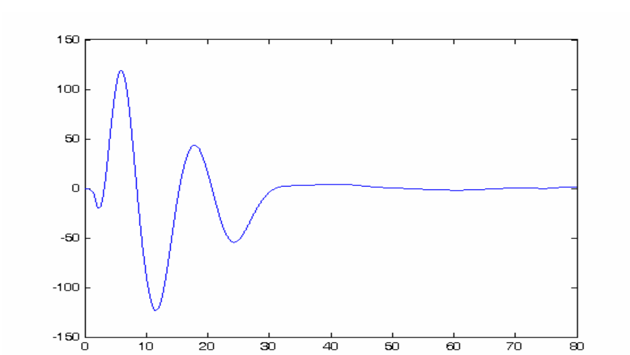

Figure 3. The model reference adaptive tracking control

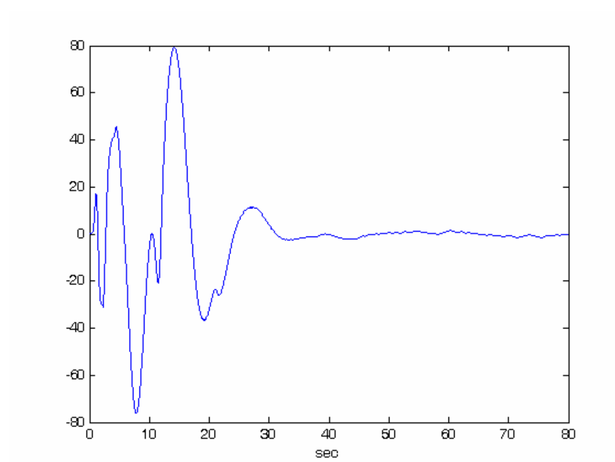

Figure 6. The model reference adaptive tracking control

5. CONCLUSIONS

In this paper a global robust model reference adaptive output feedback control is proposed for SISO nonlinear systems containing unknown constant parameters entering linearly in the state equations and subject to bounded disturbances. The main contribution of this paper is that there is no a *priori* information on the high frequency gain. Moreover, the bounded disturbances are bounded by an unknown bound. To achieve asymptotic output tracking, the proposed adaptive control algorithm uses a Nussbaum gain. Lyapunov stability criterion and the dead zone technique are applied to ensure that all the signals are globally bounded and the tracking error is arbitrary small.

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Biographies

Nabila A. S. Fawzy was born in 1976.She got her B.Sc. with distinction with honors in 1998. She received her M. Sc. in 2001 and her Ph.D. in 2006, both in control engineering from Electronics and Communications department, Faculty of Engineering, Cairo University, Egypt. Her research interests are in adaptive and nonlinear control.

Ahmed Ali Abouelsoud was born in Cairo, Egypt on January 8, 1964, He received the B. Sc., in electrical engineering in 1986, the M. Sc. in control engineering in 1990, and the Ph. D. in control engineering in 1995 all from Cairo University, Egypt. He is a member of IEEE automatic control society. He published more then 20 papers in automatic control. His research interests are in nonlinear and adaptive control and robotics.

Magdi Sadek Mahmoud has been a professor of systems engineering since 1984. He worked at different universities world-wide in Egypt, Kuwait, UAE, UK, USA, Singapore and Australia. He has given invited lectures in Venezuela, Germany, UK and USA. He has been actively engaged in teaching and research in the development of modern methodologies to computer control, systems engineering and information technology. He is the principal author of ten (11) books, nine (9) book-chapters and the author/co-author of more than 350 peer-reviewed papers. He is the recipient of two national, one regional and a university prizes for outstanding research in engineering. He is a fellow of the IEE, a senior member of the IEEE, the CEI (UK), and a registered consultant engineer of information engineering and systems (Egypt).

Rajia. I. Badr obtained her B.Sc. degree with distinction and honors in 1979 from the Electronics and Communications Department, Faculty of Engineering, Cairo University. She got her M.Sc. degree in Systems and Control Engineering in 1982. She earned her Ph.D. in Systems and Control Engineering in 1987 from Cairo University. In 1987 she was appointed as an assistant professor in the Electronics and Communications Department, Faculty of Engineering, Cairo University. She is currently an Associate Professor at the same department and her research areas include Robust Control, Neuro-Fuzzy Control, and Adaptive-Non-Linear Control.

Mohamed F. Hassan was born in Cairo, Egypt on June 21, 1947. He received the B. Sc. and M. Sc. degrees in Electrical Engineering from Cairo University, Faculty of Engineering, Cairo, Egypt in 1970 and 1973 respectively, and D. Sc. (Doctor d'Etat) in Systems Science and Automatic Control from Paul Sabatier University, Toulouse, France, 1978. He was the recipient of many outstanding research prizes and medals from Egypt and USA. Prof. Hassan is a co-author of one book, author of many book chapters and encyclopedia chapters. Moreover, he is an author and co-author of more than 118 technical papers in most of the international journals and conferences. His research activities are in the areas of large scale systems, optimization theory, stochastic control theory, estimation theory and adaptive control.

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INSTRUCTIONS FOR AUTHORS (Title is Helvetica size 18)

Author 1[a,*], Author 2[b], Author 3[c] (Helvetica size 12)

[a]Affiliation a (Helvetica size 10)

[b]Affiliation b (Helvetica size 10)

[c]Affiliation c (Helvetica size 10)

ABSTRACT (Times New Roman size 12)

An abstract, not exceeding 200 words, is required for all papers (font is Times New Roman size 9).

Keywords (Times New Roman size 12)

The author should provide a list of key words, up to a maximum of six (font is Times New Roman size 9).

1. INTRODUCTION (Times New Roman size 12)

(Body text is Times New Roman size 9). The introduction of the paper should explain the nature of the problem, previous work, purpose, and the contribution of the paper. It is assigned the number "1" and following sections are assigned numbers as needed. format. The paper must be (Letter) page size with margins of 1.25" for the top of first page, 0.75" for left, right, and bottom, and 0.75" for top, bottom, left, and right of ALL subsequent pages. The paper must be double column format, single line spacing with an extra line added between paragraphs. Please limit the title to a maximum length of 10 words. The author's name(s) follows and is also centered on the page. A blank line is required between the title and the author's name(s). Last names should be spelled out in full and preceded by author's initials. The author's affiliation, complete mailing address, and e-mail address are provided below. Phone and fax numbers do not appear. Do not underline any of the headings, or add dashes, colons, etc. The headings, starting with "1. INTRODUCTION", appear in upper case letters and should be set in bold and aligned flush left. All headings from the Introduction to Conclusions are numbered sequentially using 1, 2, 3, etc. Subheadings are numbered 1.1, 1.2, etc. If a subsection must be further divided, the numbers 1.1.1, 1.1.2, etc. are used and the number and associated title are set in italics instead of bolded. A colon is inserted before an equation is presented, but there is no punctuation following the equation. All equations are numbered and referred to in the text solely by a number enclosed in a round bracket (i.e. (2) reads as "equation 2"). Ensure that any miscellaneous numbering system you use in your paper cannot be confused with a reference [3] or an equation (2) designation.

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1.1 Initial Submission

The manuscript must be a single file (including tables, figures, etc.) using either an Adobe-compatible portable document format (*.pdf) or an MS Word (*.doc) or Adobe FrameMaker (*.fm). Submit the manuscript without the authors' names, affiliation, and biographies. Along with it, submit a cover page that includes the manuscript title, authors' names and affiliation, and the corresponding author's name and contact information (full postal and e-mail addresses, phone and fax numbers), and biographies. The corresponding author will be given a reference number assigned to the paper and it is to be used in all future correspondence. Formatting your paper correctly saves a great deal of editing time, which in turn ensures publication of papers in a timely manner. Authors submit electronic versions of their manuscript to:

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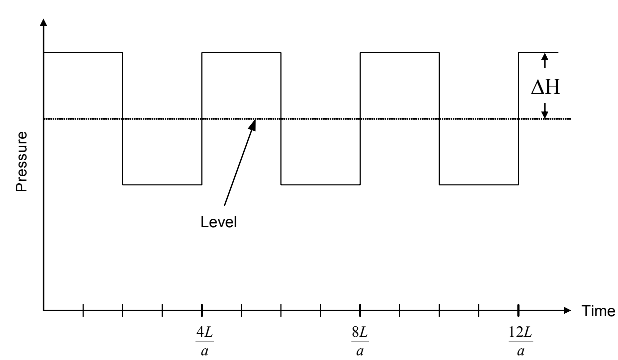

Figure 1. Sample graph

A reference list must be included. Only cited text references are included. Each reference is referred to in the text by a number enclosed in a square bracket (i.e., [3]). References must be numbered and ordered according to where they are first mentioned in the paper, not alphabetically.

REFERENCES

[1] Anderson, B. O. D. and D. J. Clements, "Algebraic Characterization", Vol. 17, 1981, pp. 703-712.

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